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Jacob Martin Jr

Louisiana State University and Agricultural & Mechanical College

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**DESIGN AND COMPARISON OF CONTROLLERS BASED ON
SIMPLIFIED MODELS OF THE PROCESS**

A Dissertation

**Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy**

in

The Department of Chemical Engineering

by

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ABSTRACT

The primary purpose of this dissertation is to study the controller design problem and to develop controller tuning relationships based on simplified models of the process which provide the control engineer some flexibility in selecting a desired response. The models utilized are restricted to the first-order-lag-plus-dead-time model and the second-order-lag-plus-dead-time model.

Controller tuning relationships based on these simplified process models are developed and presented for the controller synthesis technique and the optimal output regulator technique. Both techniques provide a parameter which can be tuned to obtain a desired closed-loop response. The controller synthesis relationships fix the integral time or integral and derivative times of a PI or PID controller as a function of the model parameters and allow for the adjustment of the controller gain to meet any specified performance criteria. This is not the case for the optimal output regulator, as the controller parameters and the tuning parameters are interrelated.

Correlations for controller tuning parameters that produce a 5% overshoot response are developed and presented as a function of the process model parameters for the controller synthesis and the optimal output regulator techniques. These correlations are then compared to the quarter-decay ratio and the integral of the absolute value of the error tuning correlations and several closed-loop

responses for a second-order-lag-plus-dead-time process are examined. The controller synthesis 5% overshoot criteria is shown to be near optimal in terms of the integral of the absolute value of the error.

The tuning correlations are then applied to a simulation of a non-linear continuous stirred tank reactor in which an exothermic second-order chemical reaction is taking place. The reactor temperature is controlled by manipulating the cooling water rate to the jacket of the reactor. The closed-loop responses for set-point and disturbance changes are presented and compared with the quarter decay ratio and the integral of the absolute value of the error responses. The unmeasured disturbance is a step change in the reactant flow rate.

CHAPTER I

INTRODUCTION

Today, most of the interest and attention in the field of automatic control theory is devoted to modern control techniques such as adaptive control, state space identification, stochastic identification and control, and optimal control of specific processes. While these topics provide the more glamorous and exciting areas of work in the field, the controller design problem, that of specifying the controller modes and tuning the controller parameters, is still a very real and important area of process control.

This is more evident today than ever before. Scarcity and high cost of raw materials, high cost of energy and resources, tighter controls on environmental standards, labor problems, and the required profit statement to stay in business have caused many industrial organizations to look more closely at control schemes, tuning, and digital computer control applications to assist them in overcoming these environmental and economic burdens.

Although much work has been done in the area of controller tuning, industrial acceptance and application has been slow and to a great extent is still done by trial and error. The most widely accepted techniques are those of Ziegler-Nichols (1), Lopez, et al. (2), and Rovira, Murrill, and Smith (3). These techniques

are based on fixed performance criteria and in most industrial applications are only used as starting points for tuning control loops.

Since it has become increasingly important to produce more product within a given specification while minimizing the use of raw materials, energy, resources, and the production of off-spec products, it is desirable to provide the control engineer with the flexibility to specify or select a desired response.

The purpose of this dissertation is to investigate controller tuning techniques which provide this flexibility and to examine their feasibility for industrial applications.

Two techniques are presented in Chapter II which provide the flexibility to obtain a desired response, controller synthesis and optimal linear regulator. A brief discussion of process models and performance criteria is also included in Chapter II.

A performance criteria is established in Chapter III to develop controller tuning relationships for the techniques. The relationships are presented in Chapters III and IV and are compared to Ziegler-Nichols and Rovira's techniques for controller synthesis and optimal linear regulator, respectively. A brief discussion of a $PIDD^2$ (proportional-plus-integral-plus-derivative-plus-derivative-squared) controller is also included in Chapter IV.

The application of these techniques to a simulation of a non-linear continuous stirred tank reactor is presented in Chapter V. Both set-point and load or disturbance changes are considered.

In summary, the purpose of this research is to develop controller tuning techniques which provide the control engineer flexibility to obtain a desired response and examine the feasibility of industrial applications for these techniques.

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CHAPTER II

DEFINITION OF THE CONTROLLER DESIGN PROBLEM

Introduction

The controller design problem consists of selecting the modes and tuning the controller parameters for a given process. Although much work has been done in the area of controller tuning, industrial acceptance and application has been slow and to a great extent is still done by trial and error. Of the current available techniques, the most widely noted and accepted are those of Ziegler-Nichols (1) who in 1942 pioneered the field by developing tuning relationships based on quarter decay ratio criteria, and in 1967, Lopez et al, (2), and in 1969, Rovira, Murrill, and Smith (3) developed tuning relationships based on the minimum integral performance criteria. These techniques are often used as starting points for tuning industrial process control loops.

Current tuning techniques are for the most part based on fixed performance criteria and do not give the control engineer the flexibility to specify percent overshoot, rise time, settling time, or any other performance criterium. This chapter presents two methods which provide the control engineer with this flexibility, controller synthesis and applied optimal regulator theory. Both methods require some prior knowledge of the process dynamics, therefore, a brief discussion of process models is also included.

Adapting the Controller to the Process

A typical process control loop is sketched in Figure 2-1. In this sketch the block labeled "Process" includes, in addition to the process transfer function, the transfer functions of the control valve, the sensor and the transmitter. This is not the oversimplification it appears to be since, in practice, when the process dynamics are experimentally determined on the plant, intermediate variables such as flow through the valve, valve position, etc. are not generally available. Only the controller output signal and transmitter signal can be measured or recorded in the typical case.

The controller design problem consists of selecting the modes and tuning the controller parameters. The simplest way of doing this is by trial and error on the actual plant. At the other extreme, computer simulation of the control loop offers a most sophisticated method for controller design. Somewhere in between, the use of simplified models of the process from an experimentally determined process reaction curve offers a compromise between tuning accuracy and expenditure of time and money. The methods presented here use this latter approach, since it is most practical for industrial use.

Process Reaction Curve Models

The process reaction curve is the time response of the transmitter output signal (controller input) to an arbitrarily applied step change in controller output and is illustrated in Figure 2-2. From this recorded response it is possible to obtain the parameters of a simplified model of the process.

FIGURE 2-1
TYPICAL PROCESS CONTROL LOOP

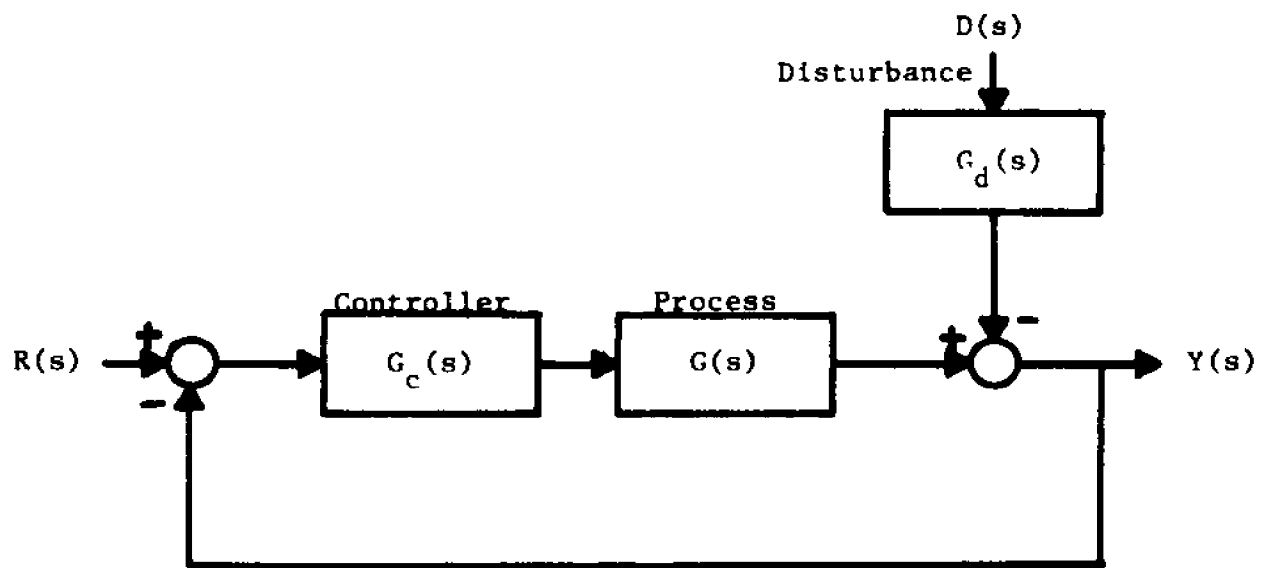
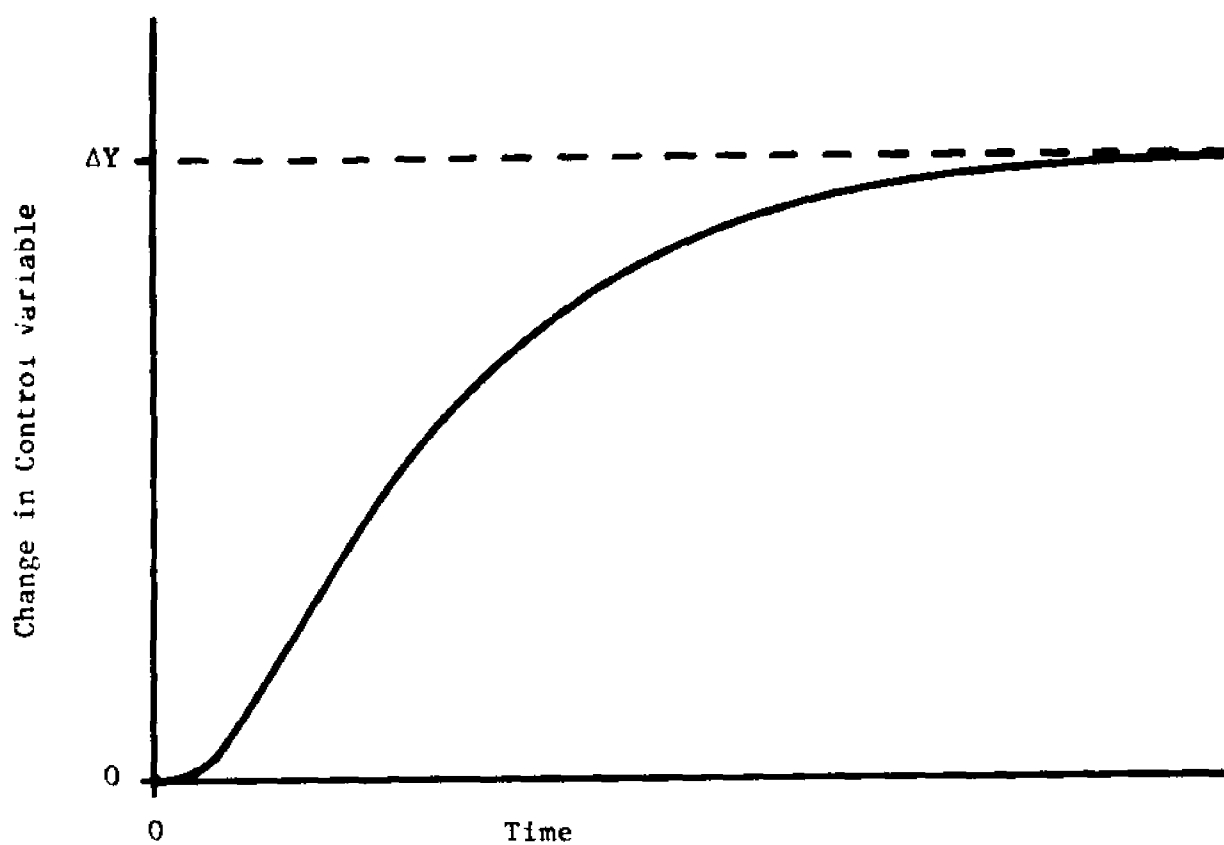


FIGURE 2-2
PROCESS REACTION CURVE



The most widely used model is that of the first-order-lag-plus-dead-time (transportation lag or time delay) model. The transfer function of this model is of the form

$$G(s) = \frac{Ke^{-t_0 s}}{\tau s + 1} \quad [2.1]$$

where $G(s)$: process transfer function

K : steady-state gain

τ : time constant

t_0 : dead-time or transportation lag

s : Laplace transform variable.

Ziegler-Nichols (1), Miller (4), and Smith (5) have proposed techniques for determining the parameters of the first-order-lag-plus-dead-time model. The techniques of Ziegler-Nichols and Miller are based on the graphical construction of a tangent line at the point of steepest slope on the process reaction curve to determine τ and t_0 . Although the graphical construction of a tangent line is conceptually simple, it is often difficult to accurately draw in practice. Smith provides an alternate technique to overcome this inaccuracy. His technique uses two points on the process reaction curve and the analytical solution of the first-order-lag-plus-dead-time transfer function for a step change in input. This results in two equations with two unknowns, τ and t_0 .

$$\begin{aligned} t \Big|_{0.284 \Delta y} &= t_0 + \frac{\tau}{3} \\ t \Big|_{0.632 \Delta y} &= t_0 + \tau \end{aligned} \quad [2.2]$$

These equations can be solved simultaneously for the model parameters.

The process gain, K , is determined as the ratio of the change in output (transmitter output signal) to the change in input (step change in controller output) for the three methods.

It should be noted here, that most controller tuning techniques are based on the first-order-lag-plus-dead-time model of the process. Although this simple model represents many chemical processes well, there are cases in which a more accurate model may be desirable. This leads to the second-order-lag-plus-dead-time model which can be represented by the following transfer function:

$$G(s) = \frac{K_c e^{-t_o s}}{s^2 + bs + c} \quad [2.3]$$

where $G(s)$: process transfer function
 K : steady-state gain
 b : damping parameter
 c : frequency parameter
 t_o : dead-time or transportation lag
 s : Laplace transform variable.

It should be noted that

$$\omega_n = \sqrt{c}$$

and

$$\zeta = \frac{1}{2} \cdot \frac{b}{\sqrt{c}}$$

[2.4]

where ω_n is the natural frequency and ξ is the damping ratio. For most processes, the denominator of equation 2.3 can be factored into two time constants, τ_1 and τ_2 . This form of the second-order-lag-plus-dead-time transfer function is

$$G(s) = \frac{K e^{-t_o s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad [2.5]$$

where τ_1 is the larger or dominant time constant and τ_2 is the smaller or secondary time constant. Equation 2.5 is related to equation 2.3 by the following:

$$\tau_1 = \frac{b + \sqrt{b^2 - 4c}}{2c}$$

and

$$\tau_2 = \frac{1}{\tau_1 c}$$

[2.6]

In some cases the process is underdamped and it is not possible to factor the denominator of equation 2.3 into two real time constants. The most typical example of an underdamped process is the one that constitutes the secondary or slave loop in a cascade control system. Graphical and numerical methods of computing the second-order-lag-plus-dead-time model parameters have been investigated by Stern (6), Oldenburg and Satorius (7), Smith (8) and Meyer (9). The work of Sten, Oldenburg and Sartorius, and Smith is restricted to overdamped systems in the factored form. Meyer's work is for overdamped and underdamped systems and is in terms of the natural frequency, ω_n , and the damping ratio, ζ .

Chiu (10) found that of the graphical techniques for the first-order-lag-plus-dead-time model, Miller's (4) technique provided the

better fit. He also found that for the second-order-lag-plus-dead-time model, Stern's (6) and Meyer's (9) techniques were best and gave similar results. Chiu's work was based on a third-order-lag-plus-dead-time process.

Methods to fit models of higher order than second-order-lag-plus-dead-time from the process reaction curve have not been developed because the step-change response does not produce enough information. More sophisticated forcing methods must be used to develop higher order models. Of these, pulse testing (22) is the only one that has proven successful in process identification.

Controller Synthesis

The controller synthesis method consists of calculating the controller transfer function $G_c(s)$ that will produce a desired closed-loop transfer function. The closed-loop transfer function of the block diagram in Figure 2.1 for a set-point input is known, from block diagram algebra, as:

$$\frac{Y(s)}{R(s)} = \frac{G_c(s) G(s)}{1 + G_c(s) G(s)} \quad [2.7]$$

where $Y(s)$: Laplace transform of transmitter signal
 $R(s)$: Laplace transform of set-point signal
 $G_c(s)$: Controller transfer function
 $G(s)$: Process transfer function.

Simple algebraic manipulation of equation 2.7 yields a formula for the direct computation of the controller transfer function:

$$G_c(s) = \frac{1}{G(s)} \frac{Y(s)/R(s)}{1 - Y(s)/R(s)} \quad [2.8]$$

In this formula $G(s)$ is the process transfer function determined from the process reaction curve or from basic principles, and $Y(s)/R(s)$ is the desired closed-loop transfer function. It is evident from equation 2.8 that the higher the order and complexity of the process transfer function the higher will be the complexity of the controller. In what follows it will be shown that if the process is represented by a first-order-lag transfer function, a proportional-plus-integral (PI) controller will result, while if the process is a second-order-lag a proportional-plus-integral-plus-derivative (PID) controller will result. Should the order of the process be higher than two, lead lag networks would have to be used in series with the controller since controllers for higher complexity than PID are not normally available.

The Dahlin Controller

Dealing mostly with the design of digital computer control algorithms, Dahlin (11) and Higham (12) proposed the following form of the closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{\lambda \cdot e^{-t_0 s}}{s + \lambda} \quad [2.9]$$

where λ is a tuning parameter. The dead-time term, which is equal to the effective process dead-time, is necessary to avoid a predictive term, i.e. negative dead-time, in the computed controller transfer function. Increasing the value of the tuning parameter λ increases the speed of the closed-loop response. The steady-state

gain is unity, assuring the absence of steady-state error or offset.

Substitution of equation 2.9 into equation 2.8 results in the following formula for the controller transfer function:

$$G_c(s) = \frac{1}{G(s)} \cdot \frac{\lambda \cdot e^{-t_0 s}}{s + \lambda \cdot (1 - e^{-t_0 s})} \quad [2.10]$$

At this point it should be noted that the term $(1 - e^{-t_0 s})$ of the controller synthesis technique represents dead-time compensation. Equations A-5 and A-6 of Appendix A show this by developing the controller transfer functions for both techniques. Dead-time compensation was first proposed by O. J. M. Smith (13) and is better known as Smith predictor or dead-time compensator. The compensator did not gain wide acceptance because of the difficulties of implementing the model dead-time with an analog circuit. However, since the advent of digital control computers it has grown in acceptance and Bakke (14) and Corripio and Smith (15) have shown significant improvements in controlling processes with dead-time with the dead-time compensator technique for digital systems.

The exponential term in the denominator of equation 2.10 will appear in the controller transfer function. Since exponential dynamic terms cannot be obtained with standard analog components, the term cannot be realized and must be approximated. This can be done by a first-order Taylor series expansion:

$$e^{-t_0 s} \approx 1 - t_0 s \quad [2.11]$$

Substitution of this equation into equation 2.10 results in

$$G_c(s) = \frac{1}{G(s)} \cdot \frac{\lambda \cdot e^{-t_0 s}}{s(1 + \lambda \cdot t_0)} \quad [2.12]$$

which is used for the design of the controller.

PI Controller

When a first-order-lag-plus-dead-time transfer function is fitted to the process reaction curve, a proportional-plus-integral or PI controller results. This is shown by substitution of equation 2.1 into equation 2.12:

$$G_c(s) = \frac{\tau s + 1}{s} \cdot \frac{\lambda}{K(1 + \lambda t_0)} \quad [2.13]$$

Formulas for the controller gain and integral time of a standard PI controller are obtained by comparison with the PI controller transfer function:

$$G_c(s) = K_c \left(1 + \frac{1}{T_1} \cdot \frac{1}{s} \right) = \frac{K_c (T_1 s + 1)}{T_1 s} \quad [2.14]$$

where K_c : controller gain = 100/P.B.

T_1 : integral or reset time

P.B.: proportional band

Therefore, from comparison of equations 2.13 and 2.14, the tuning formulas for the PI controller are:

$$K_c = \frac{\lambda \cdot \tau}{K(1 + \lambda \cdot t_0)} \quad [2.15]$$

$$T_1 = \tau$$

PID Controller

The proportional-plus-integral-plus-derivative or PID controller results from the substitution of the second-order-lag-plus-dead-time transfer function given by equation 2.3 into equation 2.12:

$$G_c(s) = \frac{s^2 + bs + c}{s} \cdot \frac{\lambda}{cK(1 + \lambda t_o)} \quad [2.16]$$

By comparison with the standard PID controller transfer function:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i} \cdot \frac{1}{s} + T_d \cdot s \right) = \frac{K_c (T_i T_d s^2 + T_i s + 1)}{T_i s} \quad [2.17]$$

where T_d is the derivative or preact time. The tuning formulas for the PID controller is:

$$\begin{aligned} K_c &= \frac{\lambda \cdot b}{cK(1 + \lambda t_o)} \\ T_i &= b/c \\ T_d &= 1/b \end{aligned} \quad [2.18]$$

Had the factored form of the second-order-lag-plus-dead-time transfer function equation 2.5 been used in the controller synthesis, the resulting tuning formulas would be:

$$\begin{aligned} K_c &= \frac{\lambda (\tau_1 + \tau_2)}{K(1 + \lambda \cdot t_o)} \\ T_i &= \tau_1 + \tau_2 \\ T_d &= \frac{\tau_1 \cdot \tau_2}{\tau_1 + \tau_2} \end{aligned} \quad [2.19]$$

The transfer function for most off-the-shelf PID controllers is of the form:

$$G'_c(s) = K'_c \left(1 + \frac{1}{T'_i} \cdot \frac{1}{s}\right) (1 + T'_d s) \quad [2.20]$$

The value of the actual controller parameters K'_c , T'_i , and T'_d can be obtained from the values given by equation 2.18 using the following formulas:

$$\begin{aligned} K'_c &= \frac{K_c}{1 + T'_d/T'_i} \\ T'_i &= 0.5 \cdot [T_1 + \sqrt{T_1 \cdot (T_1 - 4T_d)}] \\ T'_d &= \frac{T_1 \cdot T_d}{T'_i} \end{aligned} \quad [2.21]$$

It is interesting to note that equation 2.21 will result in real values only if the process transfer function is overdamped or critically damped. In these cases, the controller parameters can be expressed in terms of the process time constants as follows:

$$\begin{aligned} T'_i &= \tau_1 \\ T'_d &= \tau_2 \\ K'_c &= \frac{K_c}{1 + \tau_2/\tau_1} = \frac{\lambda \tau_1}{K(1 + \lambda \tau_o)} \end{aligned} \quad [2.22]$$

where τ_1 is usually considered to be the largest of the two time constants.

The above results illustrate that the major advantage of the Dahlin-Higham controller is that it contains the tuning parameter λ which provides some flexibility to the

user. Note that this parameter has an effect only on the controller gain, K_c , thus reducing the number of tuning parameters to the single parameter λ or equivalent K_c . This means that either parameter, λ or K_c can be adjusted to obtain a desired response. Figure 2.3 illustrates the effect of λ on the closed-loop response of a system consisting of a second-order-lag-plus-dead-time process with a PI controller. The controller was tuned by the parameters of a first-order-lag-plus-dead-time model of the process and for the values of λ indicated on the plot. The responses are for a unit step change in set-point. As the figure shows, small values of λ result in slow responses and large values of λ result in fast responses.

Optimal Regulator

The application of optimal linear regulator theory with quadratic performance criterion is another technique whereby the control engineer can achieve some flexibility in obtaining a desired response. This flexibility is attained by the specification of a parameter in the performance function which penalizes the system for excessive movement of the valve thus obtaining different responses for different values of this parameter. The technique provides a linear feedback control policy which is optimal based on a model of the process.

In optimal control, process control loops are generally illustrated as shown in Figure 2.4. This figure is basically the same as Figure 2.1 except for optimal control, the process is generally expressed in state variable form and each variable in

FIGURE 2-3

CLOSED-LOOP RESPONSE OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME
PROCESS AS A FUNCTION OF THE TUNING PARAMETER

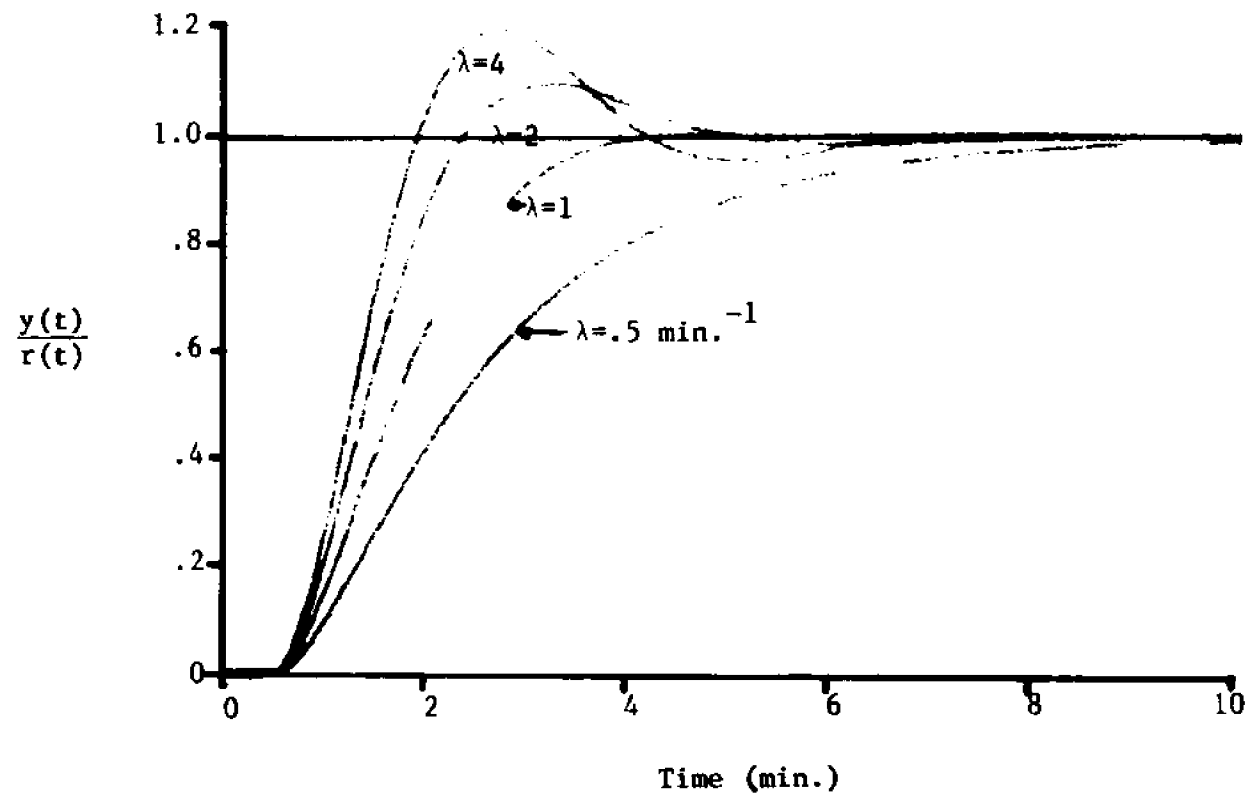
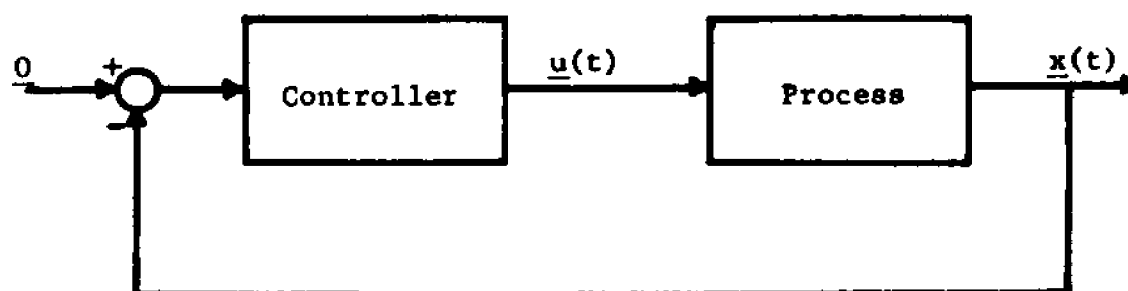


FIGURE 2-4
TYPICAL PROCESS CONTROL LOOP



the control loop has been transformed to its final or desired value. Smith (5), Athans and Falb (16), Lapidus and Luus (17), and many others illustrate the transformations necessary to cast the conventional control problem into the optimal regulator problem. If the system is linear, optimal regulator techniques can be applied to design the controller subject to the minimization of the performance function

$$I = \int_0^T [y^2 + p \dot{m}^2] dt \quad [2.23]$$

where p is the weighting factor or parameter which penalizes the system for excessive valve movement. This parameter must be specified by the designer and Figure 2.5 illustrates the effect of p on the closed-loop response.

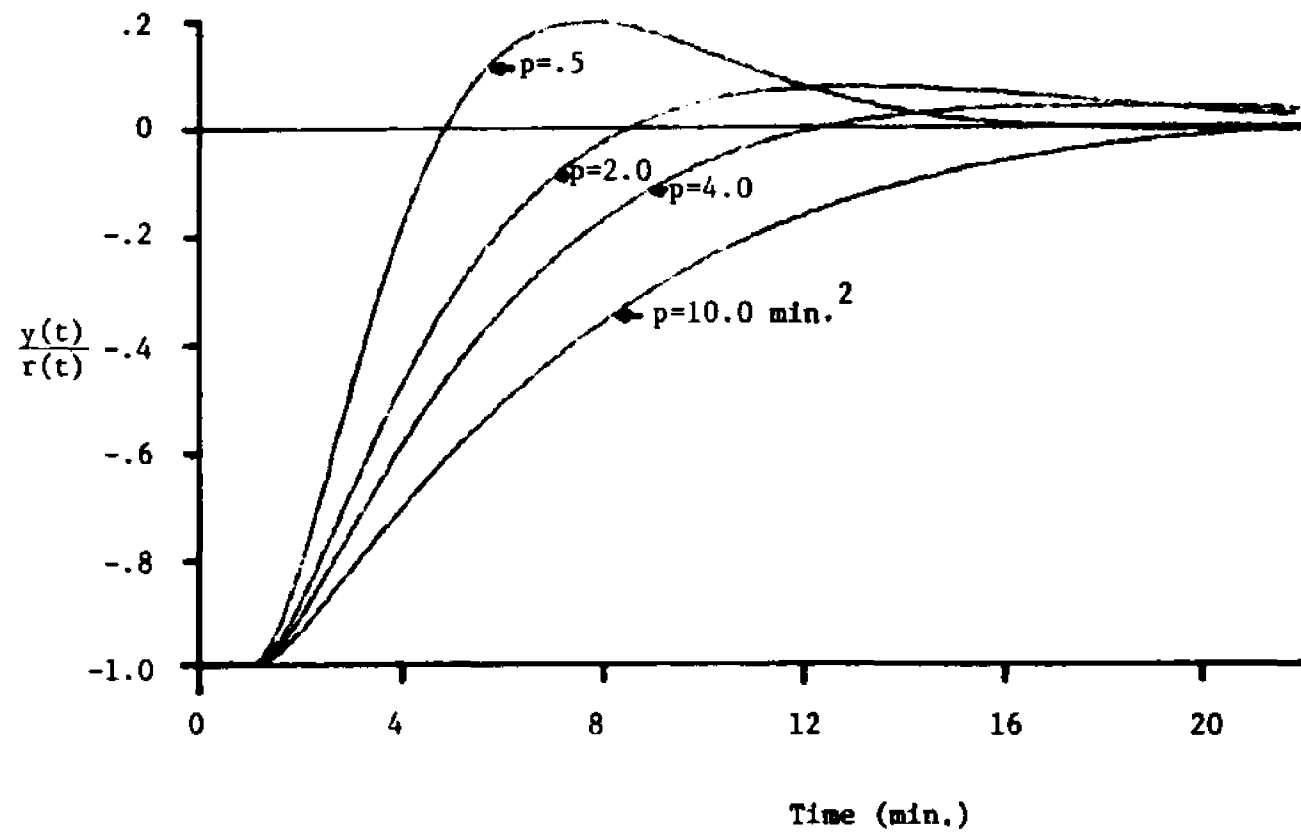
Athans and Falb have shown that for time-invariant systems, the upper limit, T , of equation 2.23 approaches infinity in the limit. This was done to guarantee that the output would stay near zero after an initial transient interval and to avoid the arbitrary specification of a large terminal time T .

It should be noted that as p approaches zero ($p \rightarrow 0$), the performance function will approach the integral of the error squared (ISE). However, the formulation of the optimal regulator does not allow p to equal zero.

Athans and Falb have chosen to classify optimal linear regulators with quadratic performance functions in two major types, state regulator (i.e., the problem of keeping the states, \underline{X} , near zero) and the output regulator (i.e., the problem of keeping the output, \underline{Y} , near zero).

FIGURE 2-5

CLOSED-LOOP RESPONSE OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME PROCESS
AS A FUNCTION OF THE TUNING PARAMETER



The work which follows falls into the class of the output regulator which is a special case of the state regulator.

Formulation of the Problem

To apply optimal regulator theory to design the controller, it will be necessary to obtain a model of the process. Lapidus and Luus (17), Miller (18) and others have illustrated the application of optimal regulator theory to extensive math-models of the process. Here the application of optimal regulator theory will be based on simplified models of the process obtained from process reaction curves. Although the use of detailed mathematical models offers more accuracy, the solution of the regulator equations increases in complexity and becomes an expensive undertaking in a major industrial operation, therefore the use of simplified models offers an alternative to the problems of expenditure, accuracy, and versatility.

The formulation of the output regulator for the linear time - invariant system is based on the minimization of the performance or cost function

$$I = \int_0^{\infty} [\underline{Y}' \underline{Q} \underline{Y} + \underline{U}' \underline{R} \underline{U}] d\sigma \quad [2.24]$$

subject to the linear process

$$\begin{aligned} \dot{\underline{X}} &= \underline{A} \underline{X} + \underline{B} \underline{U} \\ \underline{Y} &= \underline{H} \underline{X} \end{aligned} \quad [2.25]$$

where \underline{X} = state vector for the process

\underline{Y} = output vector

\underline{U} = vector of manipulated inputs to the process

$\underline{Y}' = \underline{Y}$ transpose

$\underline{U}' = \underline{U}$ transpose

The optimal control policy exists and is unique provided the following restrictions are met:

\underline{U} is not constrained

\underline{R} is positive definite

\underline{Q} is positive definite

The process is observable and controllable.

Because of the complex mathematical operations which must be performed to determine the controller parameters, the dimensionless form of the first-and second-order-lag-plus-dead-time models will be used to avoid the added complexity of dimensional analysis. The dimensionless form of these models is developed in Appendix B and the results are as follows:

1) First-order-lag-plus-dead-time model

The dimensionless form of this model is given by:

$$G(s) = \frac{K e^{-\theta s}}{s + 1}$$

where $\theta = t_o/\tau$.

The $G(s)$ or process transfer function is in dimensionless Laplace domain.

2) Second-order-lag-plus-dead-time model

The dimensionless form of this model is given by:

$$G(s) = \frac{K e^{-\theta s}}{s^2 + \beta s + 1}$$

where $\theta = t_o \sqrt{c}$

$$\beta = b / \sqrt{c} = 2\xi$$

Again the transfer function is in terms of the dimensionless Laplace domain.

The first-and second-order-lag-plus-dead-time models of the process have been transformed into state variable form in Appendix C and can be expressed as:

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} U$$

$$Y = \underline{H} \underline{X}$$
[2.26]

Note that U and Y are scalar quantities for the single-input-single-output models considered. Appendix C also shows the transformation required to express the performance function as:

$$I = \int_0^{\infty} [Y^2 + RU^2] d\sigma$$
[2.27]

Now equations 2.26 and 2.27 formulate the output regulator provided the restrictions imposed by the output regulator can be met. They are as follows:

- a) \underline{U} is not constrained

For the single-input-single-output models considered, \underline{U} is a scalar quantity and has not been constrained.

- b) \underline{R} is positive definite

For the models considered, \underline{R} is also a scalar quantity. For the first-order-lag-plus-dead-time model, \underline{R} equals $p/\tau^2 K^2$ and for the second-order-lag-plus-dead-time model, \underline{R} equals pc/K^2 . This shows that \underline{R} is a function of the tuning

parameter, p , and the parameters of the process models.

Since R is a scalar and must be positive definite ($R > 0$), this implies that p must be positive definite because τ^2 , K^2 , and c (process model parameters) are positive definite. Therefore, $p > 0$ satisfies this restriction.

c) Q is positive definite

For the single-input-single-output models considered, Q is a scalar quantity and is equal to 1.0 which is positive definite.

d) The process is observable and controllable

The tests for controllability and observability are illustrated in Appendix D for the models considered. The results show the process to be observable and controllable.

This shows that the process models considered here fulfill all the requirements of the linear time-invariant output regulator problem.

Design of Controllers

The preceding section has shown that for each process model considered the control problem can be cast into the output regulator problem for which the solution is well known, for example see Athens and Falb (16). The solution states that the optimal control policy is proportional to the states and is in linear feedback form. That is

$$\underline{U} = -\underline{K} \underline{X} \quad [2.28]$$

where $\underline{K} = \underline{R}^{-1} \underline{B}^1 \underline{J_e}$

\underline{R}^{-1} is \underline{R} inverse

\underline{B}^1 is \underline{B} transpose

$\underline{J_e}$ is the steady-state or equilibrium solution to the
Matrix Riccati Equation.

The Matrix Riccati Equation is a first-order nonlinear differential equation and is given by

$$\dot{\underline{J}} + \underline{J} \underline{A} + \underline{A}' \underline{J} - \underline{J} \underline{B} \underline{R}^{-1} \underline{B}' \underline{J} + \underline{H}' \underline{Q} \underline{H} = 0 \quad [2.29]$$

and is usually unstable in forward time. Equation 2.29 can be solved in backward time by Euler's method, since only the steady-state solution ($\dot{\underline{J}} = 0$) is needed. Only the steady-state solution is required for the time-invariant system since the upper integration limit is infinity. Thus the solution technique is a relaxation method of solving the set of nonlinear algebraic equations to obtain $\underline{J_e}$. The final condition -- which is the initial condition to the backward-time problem -- is in this sense unimportant and becomes the initial guess for the relaxation method. The solution to the output regulator problem is illustrated in block diagram form in Figure 2.6.

Now if an exact model of the process is available and the above can be performed, then the controller designed by this technique will be optimal for that process. In many cases the process models are only a mere approximation of the process or processes involved, therefore the control policy may be something less than optimal.

If the process reaction curve is fitted with a first-order-lag-plus-dead-time model and the dead-time is approximated by the

first-order Taylor series expansion

$$e^{-\theta s} = 1 - \theta s \quad [2.30]$$

the resulting state variable representation of the process model is:

$$\dot{\underline{X}} = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \underline{X} + \begin{pmatrix} -\theta \\ 1 \end{pmatrix} U \quad [2.31]$$

$$Y = [1 \ 0] \underline{X}$$

where $X_1 = Y$

$$X_2 = Km$$

$$U = K\dot{m}$$

The performance function is given by

$$I = \int_0^{\infty} [Y^2 + RU^2] d\sigma \quad [2.32]$$

where $R = p/K^2\tau^2$

The detailed derivation leading to equations 2.31 and 2.32 is presented in Appendix C. Using the controller design technique outlined in the previous section results in a controller of the form:

$$G_c(s) = K_c \left[1 + \frac{1}{T_i} \cdot \frac{1}{s} \right]$$

or a proportional-plus-integral (PI) controller. The controller parameters are defined by the model parameters and the steady-state solution of the Riccati equation by the following expressions:

$$K K_c = \frac{-\theta J e_{12} + J e_{22}}{R + \theta(-\theta J e_{12} + J e_{22})} \quad [2.33]$$

$$\frac{\tau}{T_1} = \frac{-\theta J e_{11} + J e_{21} - \theta J e_{12} + J e_{22}}{-\theta J e_{12} + J e_{22}}$$

The detailed derivations resulting in these equations are presented in Appendix E.

Replacing the dead-time with the first-order Padé approximation

$$e^{-\theta s} = \frac{2 - \theta s}{2 + \theta s} \quad [2.34]$$

increases the order of the system and the resulting state variable form of the process model is:

$$\dot{\underline{X}} = \begin{pmatrix} 0 & 1 & -1 \\ -\frac{2}{\theta} - \frac{\theta+2}{\theta} & \frac{\theta+4}{\theta} & 0 \\ 0 & 0 & 0 \end{pmatrix} \underline{X} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} U \quad [2.35]$$

$$Y = [1 \quad 0 \quad 0] \underline{X}$$

where $X_1 = Y$

$$X_2 = \dot{Y} + Km$$

$$X_3 = Km$$

$$U = Km$$

A detailed derivation of this equation is presented in Appendix C. The performance function is the same as equation 2.32. The controller resulting from this model is of the form:

$$G_c(s) = K_c \left[1 + \frac{1}{T_1} \cdot \frac{1}{s} + T_d \cdot s \right]$$

or a proportional-plus-integral-plus-derivative (PID) controller.

The controller parameters are as follows:

$$\begin{aligned}
 K K_c &= \frac{(\theta + 4) J_{e32} + (\theta + 2) J_{e33}}{\theta J_{e32} + \theta J_{e33} + 2R} \\
 \frac{\tau}{T_i} &= \frac{2(J_{e31} + J_{e32} + J_{e33})}{(\theta + 4) J_{e32} + (\theta + 2) J_{e33}} \\
 \frac{T_d}{\tau} &= \frac{\theta (J_{e32} + J_{e33})}{(\theta + 4) J_{e32} + (\theta + 2) J_{e33}}
 \end{aligned} \tag{2.36}$$

Detailed derivations of these equations are presented in Appendix E.

If the process reaction curve is fitted with a second-order-lag-plus-dead-time model and the dead-time is approximated by the first-order Taylor series expansion

$$e^{-\theta s} \approx 1 - \theta s$$

the resulting state variable representation of the process model is:

$$\dot{\underline{X}} = \begin{bmatrix} 0 & 1 & -\theta \\ -1 & \beta & 1+\theta\beta \\ 0 & 0 & 0 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = [1 \quad 0 \quad 0] \underline{X}$$

where

$$\begin{aligned}
 X_1 &= Y \\
 X_2 &= \dot{Y} + \theta K_m \\
 X_3 &= K_m \\
 U &= K_h \ddot{h}
 \end{aligned}$$

The performance function is given by

$$I = \int_0^{\infty} [Y^2 + RU^2] d\sigma \quad [2.38]$$

where $R = pc/K^2$

The detailed derivations leading to these equations are presented in Appendix C. The controller resulting from the use of this process model is of the form:

$$G_c(s) = K_c \left[1 + \frac{1}{T_1} \cdot \frac{1}{s} + T_d \cdot s \right]$$

or a PID controller. The controller parameters are as follows:

$$\begin{aligned} KK_c &= \frac{(1 + \beta\theta) J_{e32} + \beta J_{e33}}{R + \theta^2 J_{e32} + \theta J_{e33}} \\ \frac{1}{\sqrt{c} T_1} &= \frac{J_{e31} + \theta J_{e32} + J_{e33}}{(1 + \beta\theta) J_{e32} + \beta J_{e33}} \\ \sqrt{c} T_d &= \frac{\theta J_{e32} + J_{e33}}{(1 + \beta\theta) J_{e32} + \beta J_{e33}} \end{aligned} \quad [2.39]$$

Detailed derivations of these equations are presented in Appendix E.

Replacing the dead-time with the first-order Padé approximation

$$e^{-\theta s} = \frac{2 - \theta s}{2 + \theta s}$$

increases the order of the system and the resulting state variable representation of this process model is

$$\dot{\underline{X}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -\frac{2}{\theta} & -(\frac{2\beta}{\theta}+1) & -(\frac{2}{\theta}+\beta) & (\frac{4}{\theta}+\beta) \\ 0 & 0 & 0 & 0 \end{pmatrix} \underline{X} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} U \quad [2.40]$$

$$Y = [\quad 1 \quad \quad 0 \quad \quad 0 \quad \quad 0 \quad] \underline{X}$$

where

$$\begin{aligned} X_1 &= Y \\ X_2 &= \dot{Y} \\ X_3 &= \ddot{Y} + Km \\ X_4 &= Km \\ U &= K\ddot{h} \end{aligned}$$

A detailed derivation of this equation is presented in Appendix C. The performance function is the same as equation 2.38. The controller resulting from this process model is of the form:

$$G_c(s) = K_c \left[1 + \frac{1}{T_1} \cdot \frac{1}{s} + T_{d1} \cdot s + T_{d2} \cdot s^2 \right]$$

or a proportional-plus-integral-plus-derivative-plus-derivative squared (PIDD²) controller. The controller parameters are defined by the following:

$$KK_c = \frac{J e_{42} + (\beta + \frac{\theta}{2}) J e_{43} + (\beta + \frac{\theta}{2}) J e_{44}}{R + \frac{\theta}{2} J e_{43} + \frac{\theta}{2} J e_{44}}$$

$$\begin{aligned}
\frac{1}{\sqrt{c} \cdot T_1} &= \frac{J e_{41} + J e_{43} + J e_{44}}{J e_{42} + (\beta + \frac{\theta}{2}) J e_{43} + (\beta + \frac{\theta}{2}) J e_{44}} \\
\sqrt{c} \cdot T_{d1} &= \frac{(2 + \frac{\theta}{2}\beta) J e_{43} + (1 + \frac{\theta}{2}\beta) J e_{44}}{J e_{42} + (\beta + \frac{\theta}{2}) J e_{43} + (\beta + \frac{\theta}{2}) J e_{44}} \\
c \cdot T_{d2} &= \frac{\frac{\theta}{2} J e_{43} + \frac{\theta}{2} J e_{44}}{J e_{42} + (\beta + \frac{\theta}{2}) J e_{43} + (\beta + \frac{\theta}{2}) J e_{44}}
\end{aligned} \tag{2.41}$$

The detailed derivations of these equations are presented in Appendix E.

At this point, one should recognize that the PI, PID, and PIDD² controller parameters are very complex functions of the process model parameters and the tuning parameter (or penalty parameter) p . It should also be noted that the Matrix Riccati equation must be solved for each value of p for each case in order to determine the value of the controller tuning parameters.

Although regulator theory offers some flexibility in tuning controllers, its mathematical complexities certainly have been the major factor in its lack of acceptance and application in industry. However, some work has been done (other than the application to extensive mathematical models) in attempting to adapt it for industrial use. Pusch (19) developed tuning relationships for a PID controller based on the first-order-lag-plus-dead-time model of the process with Padé approximation for the dead-time. His relationships were developed for a constant step change in load or disturbance, which was a concept introduced by Johnson (20). O'Connor and Denn (21), compared

the optimal regulator, Ziegler-Nichols, and Cohen-Coon controller design techniques for the first-order-lag-plus-dead-time model. The results indicate that for processes which can be represented by such a linear model, there was extraordinarily good agreement between the optimal control and the classical settings. However, the work was limited to one model and for a small range of the penalty function.

Comparison Criteria

In order to judge the performance of controllers for different models, it is necessary to establish some basis for comparison.

In the early 1940's, quarter decay ratio was introduced by Ziegler-Nichols (1) as the criteria for tuning controllers. In the late 1960's, integral criteria was introduced by Lopez et al. (2) and Rovira, Murrill, and Smith (3), as the criteria for tuning controllers. These integral criteria were as follows:

- 1) Minimum integral of the error squared (ISE)

$$ISE = \int_0^{\infty} e^2 dt$$

- 2) Minimum integral of the absolute error (IAE)

$$IAE = \int_0^{\infty} |e| dt$$

- 3) Minimum integral of the time weighted absolute error (ITAE)

$$ITAE = \int_0^{\infty} t |e| dt$$

In addition to these, regulator theory provides for the

minimization of a quadratic performance function I , given by equation 2.24. Although the size or value of ISE, IAE, ITAE, or I may be a measure of the overall system performance, it provides very little information on the system's transient response. Because this response is important, the following will be used in the basis for comparison: rise time, settling or response time, and percent overshoot.

These terms are illustrated in Figure 2.6 and are typically defined as follows:

- a) The rise time is the time necessary for the control variable to initially reach its desired value.
- b) The settling or response time is the time necessary for the control variable to come within ± 1 or ± 5 percent of its desired value and remain between these limits.
- c) The percent overshoot is the percentage by which the control variable exceeds the desired value $(e_2/e_1)100$.

Generally a reduction in rise time can usually be obtained only at the expense of increase overshoot and settling time and a less stable system.

Set-point versus Disturbance Change

Although the transfer function (system response) for a disturbance is different than the transfer function (system response) for a set-point change, tuning a controller for a unit-step change in set-point considers the worst case for disturbance inputs. That is, the worst case for disturbance input is a step-change in the control variable with no lags or dead-time as illustrated in Figure 2.7.

FIGURE 2-6

RESPONSES TO A SET-POINT CHANGE

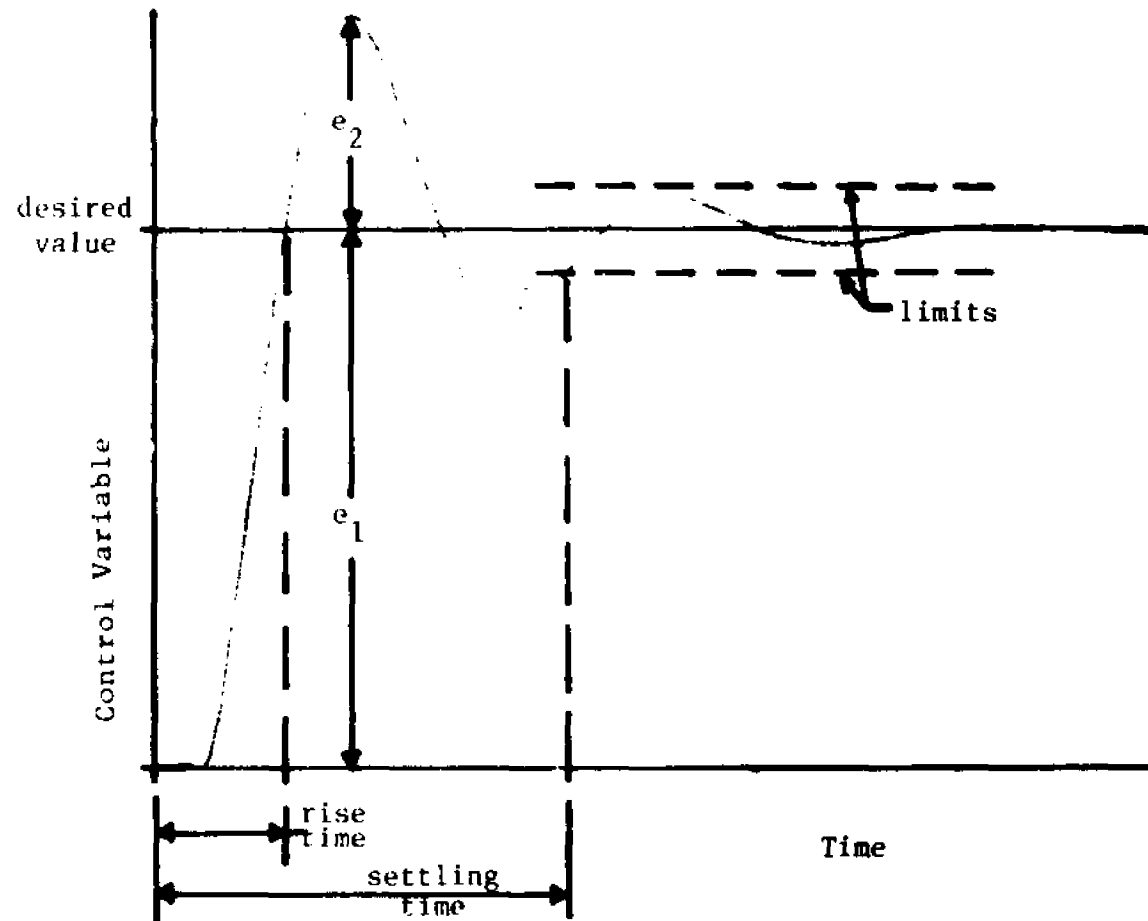
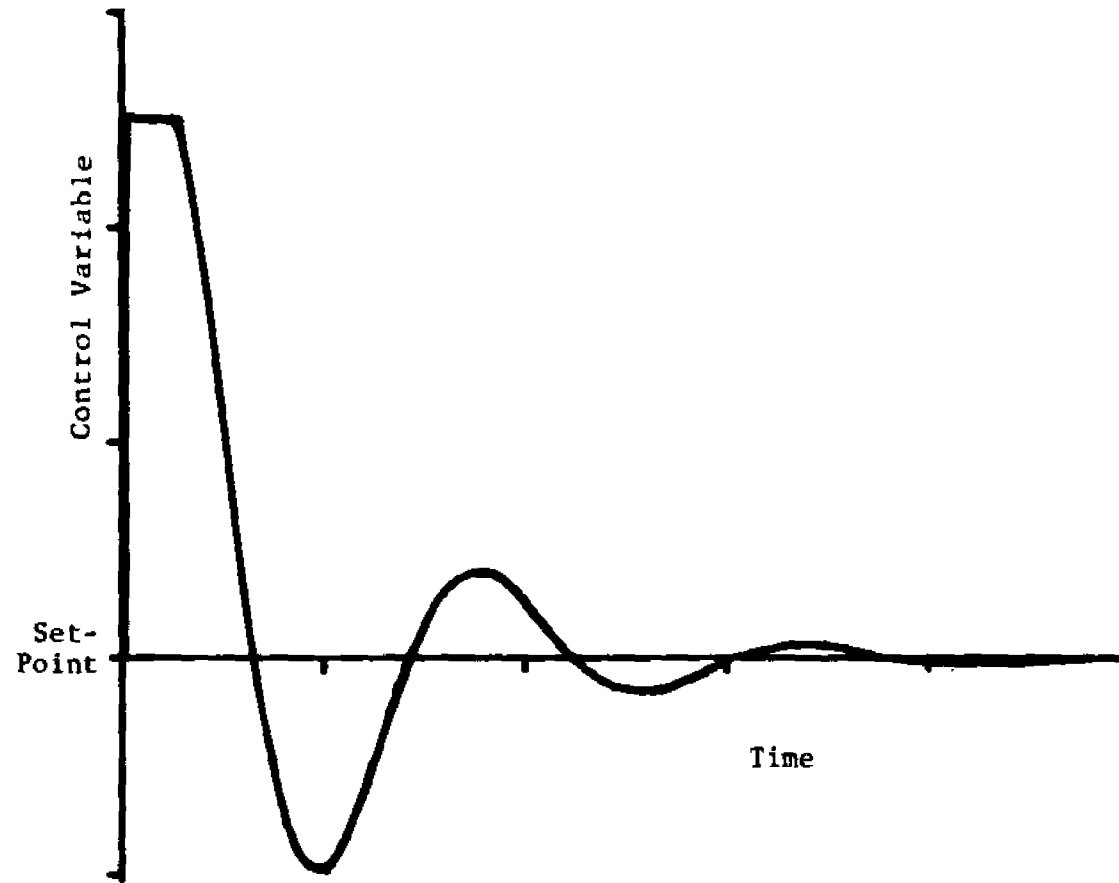


FIGURE 2-7

RESPONSE TO A DISTURBANCE CHANGE



From the block diagram in Figure 2.1 the transfer function for disturbance inputs is

$$\frac{Y(s)}{D(s)} = \frac{G_d(s)}{1 + G_c(s) G(s)} \quad [2.42]$$

The controller synthesis design equation is determined by solving this equation for $G_c(s)$.

$$G_c(s) = \frac{G_d(s) D(s) - Y(s)}{G(s) Y(s)} \quad [2.43]$$

Now, consider the worst case for disturbance inputs is given by

$$G_d(s) D(s) = \frac{1}{s}$$

and the desired response $Y(s)$ is given by

$$Y(s) = \frac{1 - e^{-t_0 s}}{s} + \frac{e^{-t_0 s}}{s + \lambda}$$

Substituting this into equation 2.43 results in the following controller equation

$$G_c(s) = \frac{1}{G(s)} \frac{\lambda e^{-t_0 s}}{s + \lambda(1 - e^{-t_0 s})} \quad [2.44]$$

Note, this equation is identical to equation 2.10 developed for a step-change in set-point. Therefore, controller synthesis for set-point changes include the worst case for disturbance changes.

In the optimal regulator, this is easier to see since the variables are transformed to their final states, and the responses to set-point changes and to the worst disturbance input would be the same.

Therefore only set-point changes need to be considered in the development of tuning relationships.

Summary

This chapter has presented the development of two controller design techniques which offer the control engineer some flexibility in selecting a desired response. Of the two techniques, controller synthesis seems to be the most promising from an industrial standpoint because it is simpler and reduces the number of tuning parameters to one, the controller gain.

Both techniques are based on simplified process models obtained from a process reaction curve. Therefore a brief discussion of process models and the process reaction curve was included.

Also included was a short discussion on comparison criteria and set-point versus disturbance tuning.

The next two chapters will develop the tuning relationships needed to make the techniques a useful tool for industrial use. They will also contain comparisons to current tuning relationships.

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CHAPTER III

CONTROLLER SYNTHESIS CORRELATIONS AND RESULTS

Introduction

The main advantage of the Dahlin(1)-Higham(2) type controller is the flexibility it provides the control engineer to select a desired response. Along with this flexibility, the most attractive feature from an industrial or application standpoint is that it reduces the number of tuning parameters to one. As shown by equations 2-15 and 2-18 in Chapter II, the tuning parameter λ appears only in the controller gain. Thus T_i or T_i and T_d are determined by the process dynamics and only K_c , the controller gain, need to be adjusted to tune a process control loop containing a PI or PID controller.

The effect of the tuning parameter λ on the closed-loop response of the system is illustrated in Figure 3-1. The system consists of a second-order-lag-plus-dead-time process with a PID controller. The process parameters are as follows:

$$K \equiv \text{process gain} = 1.0$$

$$b \equiv \text{process parameter} = 4.0 \text{ [time}^{-1}\text{]}$$

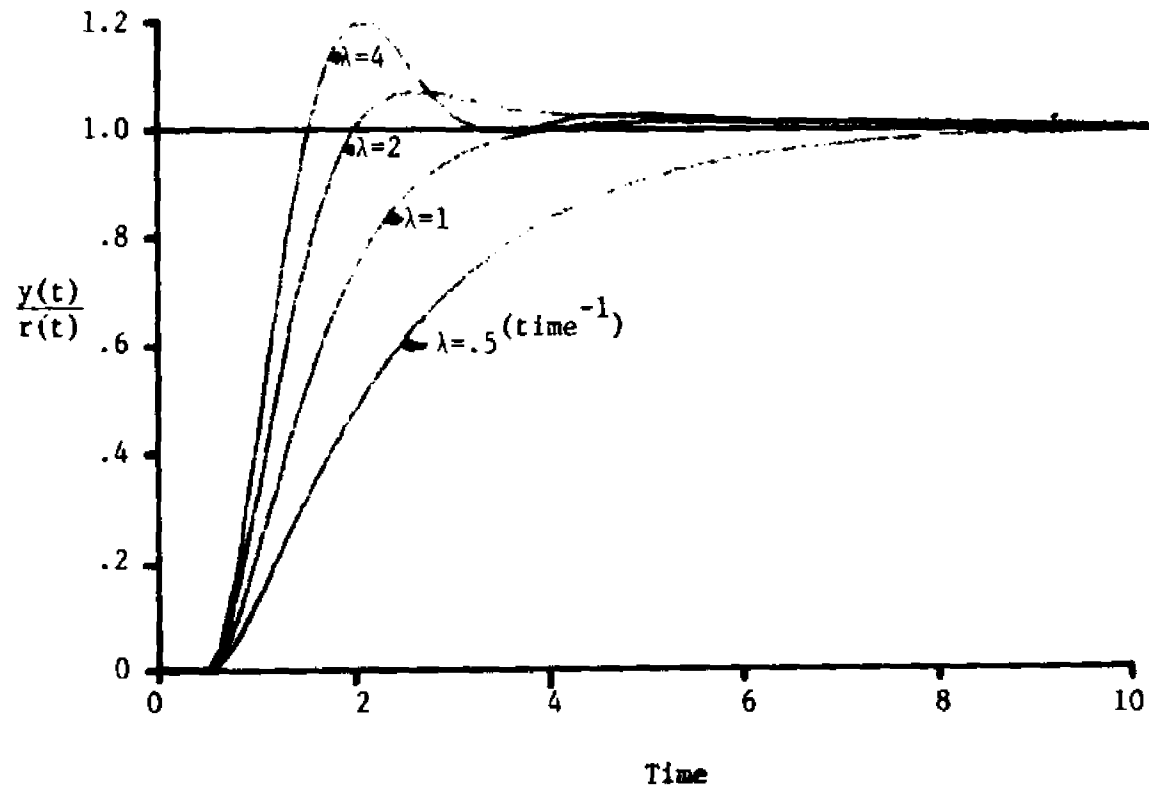
$$c \equiv \text{process parameter} = 1.0 \text{ [time}^{-2}\text{]}$$

$$t_0 \equiv \text{process dead-time} = 0.5 \text{ [time]}$$

The values of λ are indicated on the plot and have the units of time^{-1} . As can be seen from Figure 3-1, the closed-loop response is sluggish for small values of the tuning parameter, λ , but as λ is increased, the system responds faster. However, the fast

FIGURE 3-1

CLOSED-LOOP RESPONSE OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME PROCESS WITH A
PID CONTROLLER AS A FUNCTION OF THE TUNING PARAMETER



response obtained by increasing λ beyond some value, is at the expense of large overshoot. If the tuning equations are to be of any use, it is apparent that the tuning parameter must be correlated to the parameters of the process model. In order to do this, some performance criteria such as rise time, % overshoot, or minimum integral of an error function must be established. Defining the correct performance criteria is one of the most difficult task in process control, both in theory and in application.

The purpose of this chapter is to present the necessary correlations to make the controller synthesis results useful as a tool for tuning industrial process control loops. It also illustrates the flexibility which it provides the user to develop his own tuning relationships for a specific performance criteria of his own choosing.

Performance Criteria

A discussion of various performance criteria was provided in Chapter II. The control engineer may select one of these criteria which he feels fits his situation or he may develop his own criteria. However, since a performance criteria is necessary to develop some tuning relationships, a criteria will be chosen here based on the following arguments.

As mentioned earlier, it has become increasingly important to manufacture products within very rigid specifications. This concept gives rise to the term settling time, the time required for the control variable to reach and remain within some specified limits of the set-point or desired value, usually a value between ± 1 percent to ± 5 percent of the desired value. Products manufactured

outside these limits are considered as off-spec products and usually must be blended with higher quality product to make the desired product, recycled or reprocessed, sold as an off-spec product usually at a cheaper price, or sometimes even dumped or burned in a flare. Because of the high cost of this type of activity, it may be desirable to develop tuning relationships for 1 percent and 5 percent overshoot criteria. This concept is illustrated in Figure 3-2 which shows the closed-loop responses of a system, consisting of a second-order-lag-plus-dead-time process and a PI controller tuned by three different techniques. The second-order-lag-plus-dead-time process parameters are as follows:

$$K = \text{process gain} = 1.0$$

$$b = \text{process parameter} = 4.0 \text{ min.}^{-1}$$

$$c = \text{process parameter} = 1.0 \text{ min.}^{-2}$$

$$t_0 = \text{process dead-time} = 1.0 \text{ min.}$$

This corresponds to a natural frequency of 1.0 min.^{-1} and a damping ratio of 2. The responses are for a unit step change in the set-point. The curves are labeled A, B, and C and are the responses obtained by tuning the PI controller by the following criteria:

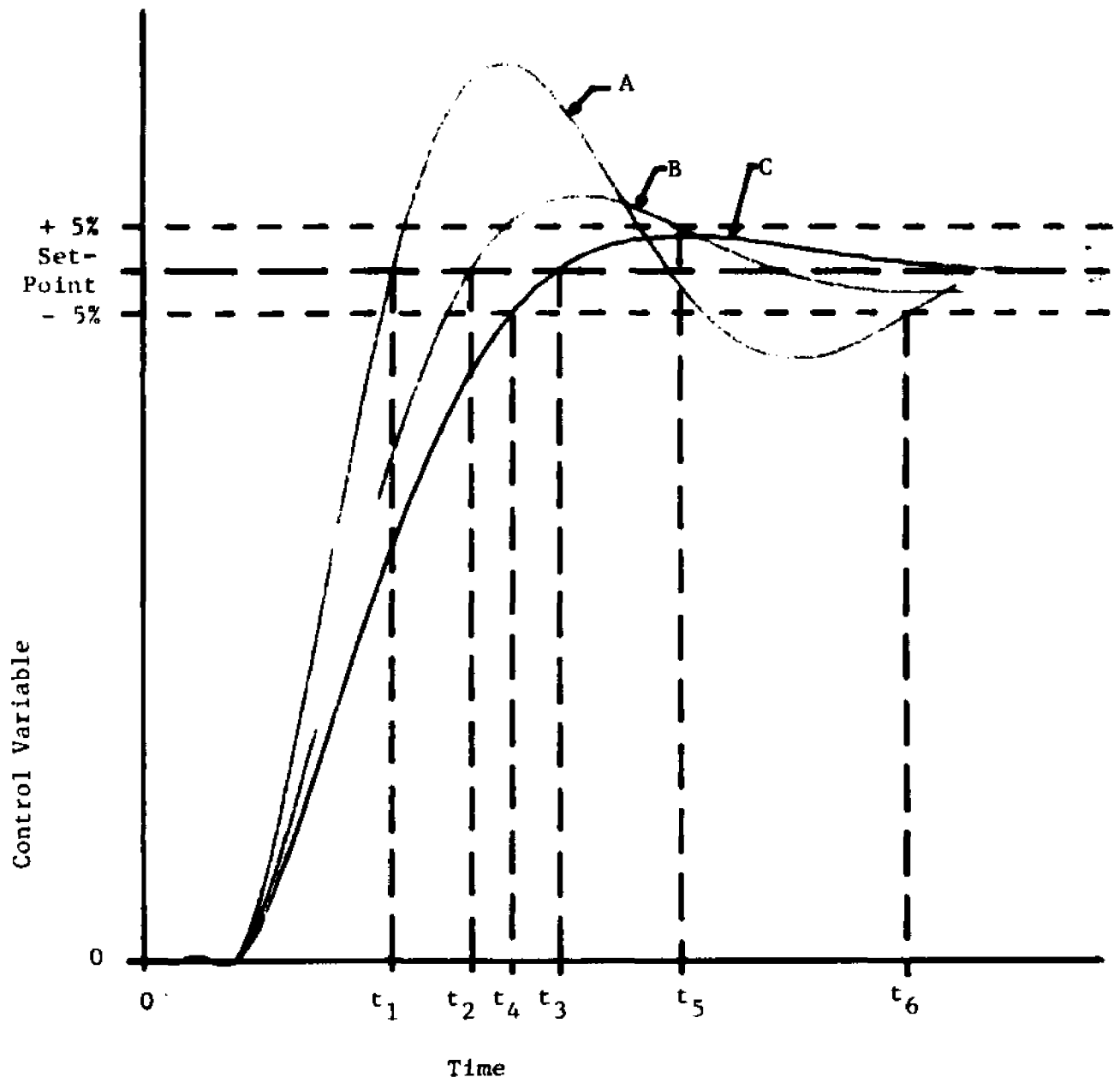
Curve A: Ziegler-Nichols (3), quarter decay ratio

Curve B: Rovira (4), minimum integral of the absolute value of the error (IAE)

Curve C: Synthesis technique, five percent overshoot

It should be noted, that all the above techniques utilize the parameters of the first-order-lag-plus-dead-time model of the process to tune the PI controller. The parameters of the process model are:

FIGURE 3-2
TYPICAL CLOSED-LOOP RESPONSE TO
STEP CHANGES IN SET-POINT



K = process gain = 1.0

τ = first-order time constant = 3.73 min.

t_0 = model dead-time or transportation lag = 1.28 min.

The tuning parameters for the PI controllers are presented in Table 3-1. As shown in Figure 3-2, curve A, quarter-decay ratio, has the fastest rise time, t_1 , but has the largest overshoot (30.7%) and the longest settling time, t_6 . Curve B, minimum IAE, has a rise time of t_2 and a settling time of t_5 and has an 11.2% overshoot. Note, curve C, five percent overshoot, has the slowest rise time t_3 , but has the fastest settling time, t_4 , which actually occurs before the rise time. The value of the IAE for the three curves are 2.92, 2.62, and 2.73 respectively. While these numbers represent the integral of the absolute value of the error, it is difficult to transfrom this in terms of product or cost other than to say that Curve B is the minimum IAE. Curves A and C are respectively, 11.5% and 4.2% larger than the minimum. However, from Figure 3-2 it can be seen that, when product quality is directly related to the controlled variable, quarter-decay ratio would produce virtually all off-spec products until time t_6 . At this point it enters the specification band and remains within these limits. Minimum IAE is similar except it does not take as long to settle into the band. The 5% overshoot criteria begins producing products to specification at time t_4 , which may be significant when considering today's high production rates and the high cost of energy and resources. For this reason, 1% and 5% overshoot were chosen as performance criteria for this work.

Process and Process Models

Now that a performance criteria has been selected, it is necessary to determine a suitable representation for the process.

TABLE 3-1

PI CONTROLLER SETTINGS

Curve	Controller Gain K_c	Reset time T_i (min.)
A quarter decay ratio	2.62	4.27
B minimum IAE	1.90	4.10
C 5% overshoot	1.51	3.73

Since a second-order-lag-plus-dead-time transfer function is considered to adequately represent most self-regulating processes, it will be used to represent the process. Only in the case of nonminimal phase systems are additional terms usually required to represent the process. Since these are very special cases and do not occur frequently, they will not be considered in this study.

The process parameters which will be considered will range from the critically damped ($\zeta = 1.0$ or $b/\sqrt{c} = 2.0$) case to the highly overdamped ($\zeta = 6.0$ or $b/\sqrt{c} = 12.0$) case. The dead-time will range from values of $t_0 \cdot \sqrt{c}$ of 0.25 to 2.0. Additional cases may be used to illustrate or examine certain aspects of the techniques and if used, will be noted as a special case.

Process models will be primarily as described in Chapter II. The first-order-lag-plus-dead-time model parameters will be obtained by Smith's (5) two-point technique. Since the process parameters are known, this technique can be applied to the second-order-lag portion of the process transfer function for which the analytical solution is well known and published, Coughanowr and Koppel (6), and others, to obtain τ , the time constant of the first-order-lag and Δ , the effective dead-time due to the second-order-lag process. The model dead-time, t_0 , is then calculated as the sum of Δ and the process dead-time. The two-point technique was chosen because it gives reproducible results which are otherwise difficult to obtain from other techniques requiring graphical constructions.

The subroutine FOPDTM, listed in Appendix G, utilizes the Secant Method to determine t @ $y = 0.284 \Delta y$ and @ $y = 0.632 \Delta y$ from the analytical solution at a second-order transfer function.

FIGURE 3-3

PARAMETERS OF THE FIRST-ORDER MODEL DUE TO A SECOND-ORDER-LAG PROCESS

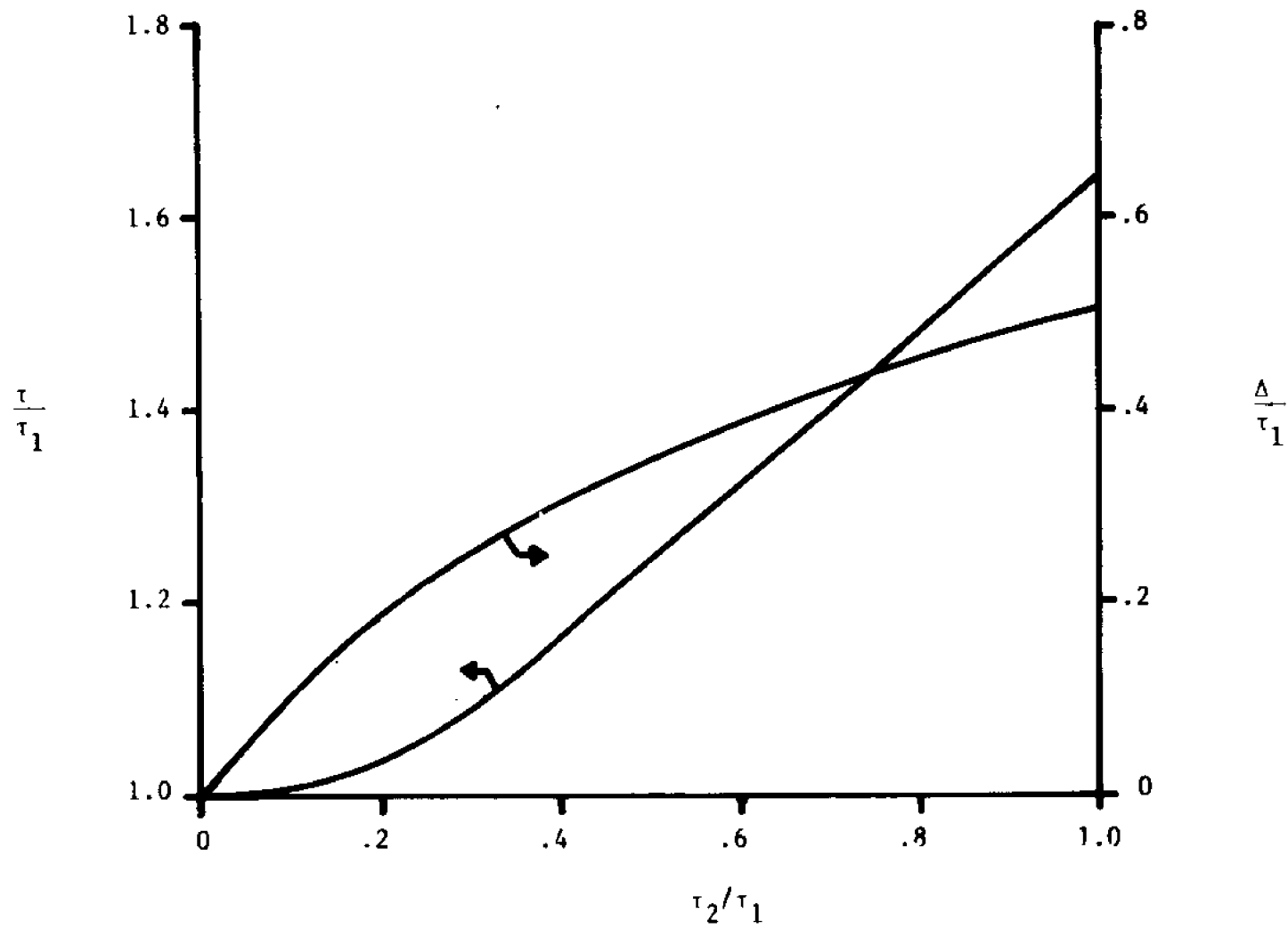
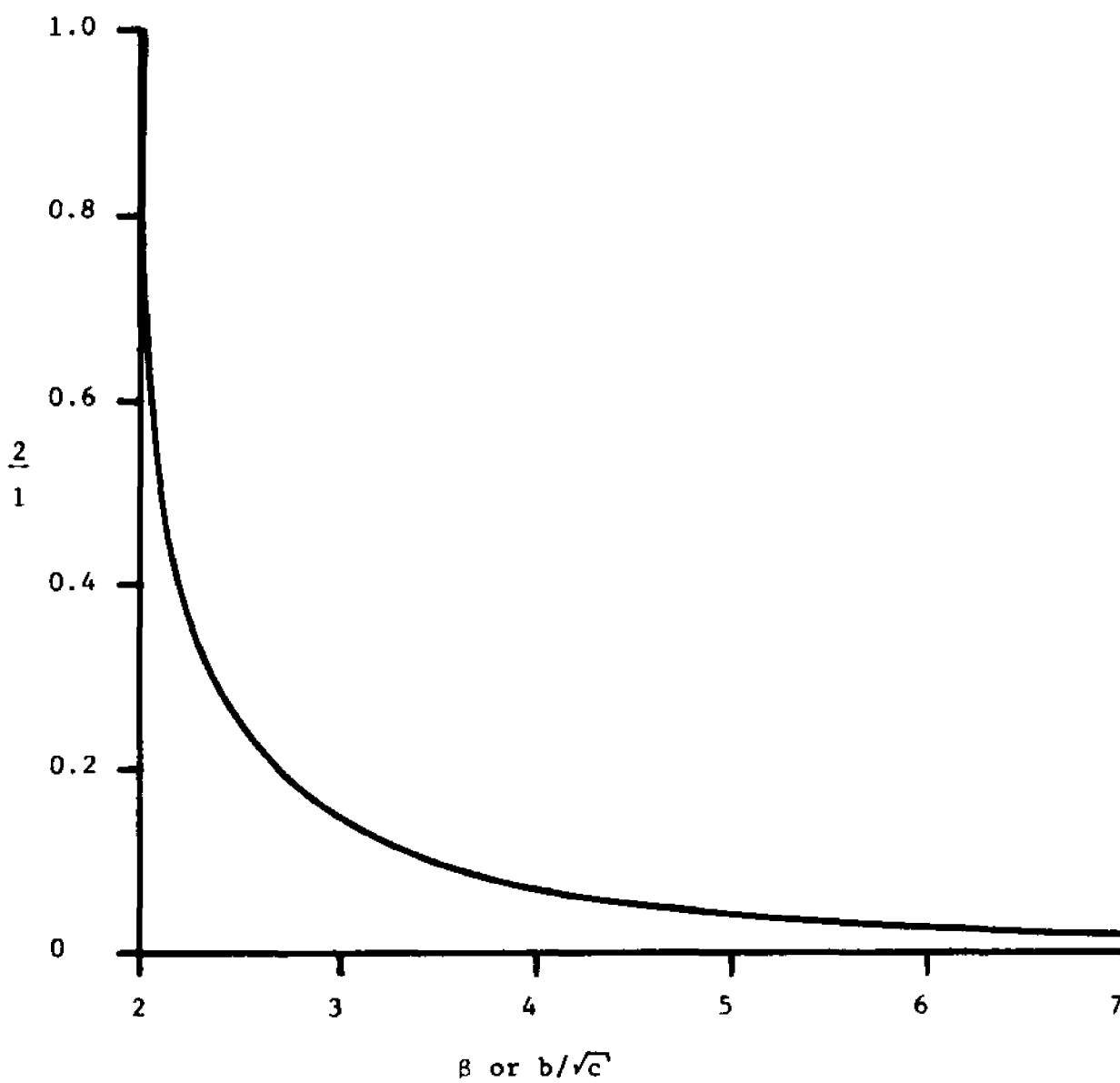


FIGURE 3-4
RATIO OF THE SECOND-ORDER-LAG TIME CONSTANTS AS A
FUNCTION OF THE DIMENSIONLESS PARAMETER OF
THE SECOND-ORDER SYSTEM



These two values of t can then be used with the following equations

$$\begin{aligned} t/0.284 \Delta y &= \Delta + \frac{\tau}{3} \\ t/0.632 \Delta y &= \Delta + \tau \end{aligned} \quad [3-1]$$

to determine τ and Δ . The results are presented as a function of the ratio of the second-order-lag-time constants in Figure 3-3. τ_1 is considered to be the larger or dominate time constant. The figure illustrates that the first-order-lag-time constant, τ , approaches τ_1 , the larger or dominate time constant and the effective dead-time Δ , approaches τ_2 , the smaller of the second-order-lag time constants, as the ratio of the second-order-lag time constants, τ_2/τ_1 , approach 0.1. The relationship of the ratio of the second-order-lag time constants to the dimensionless parameter, b/\sqrt{c} , is shown in Figure 3-4. Remembering that the damping ratio ζ is one-half the dimensionless quantity b/\sqrt{c} , this figure shows that the ratio of the two time constants goes from 1.0 to 0.1 as the damping ratio ζ goes from 1.0 (critically damped) to 1.75 or b/\sqrt{c} goes from 2.0 to 3.5. Therefore, for most over-damped systems (system's whose damping ratio is greater than 1.7) the equivalent dead-time, Δ , is simply accounting for the smaller time constant, τ_2 .

The parameters of the second-order-lag-plus-dead-time model will be the parameters of the process, since they are known and assumed to be in the form of a second-order-lag-plus-dead-time transfer function. This however will not be the case in practice, and it will be necessary to determine these parameters by one of the methods suggested in Chapter II or if a computer technique is desirable least squares or some other suitable regression method.

Since a performance criteria has been established and the process

representation has been defined, it is now necessary to determine the appropriate value of λ , the tuning parameter, which will produce the desired response.

Results for PI Controller

When the process reaction curve is fitted with a first-order-lag-plus-dead-time model, a proportional-plus-integral (PI) controller will result. This was shown in Chapter II and the resulting tuning equations for the PI controller are summarized in Table 3-2. As indicated in the table, the reset time T_1 should be set equal to τ , the first-order model time constant, and the controller gain K_c is a function of the tuning parameter λ and the model parameters τ , t_o , and K .

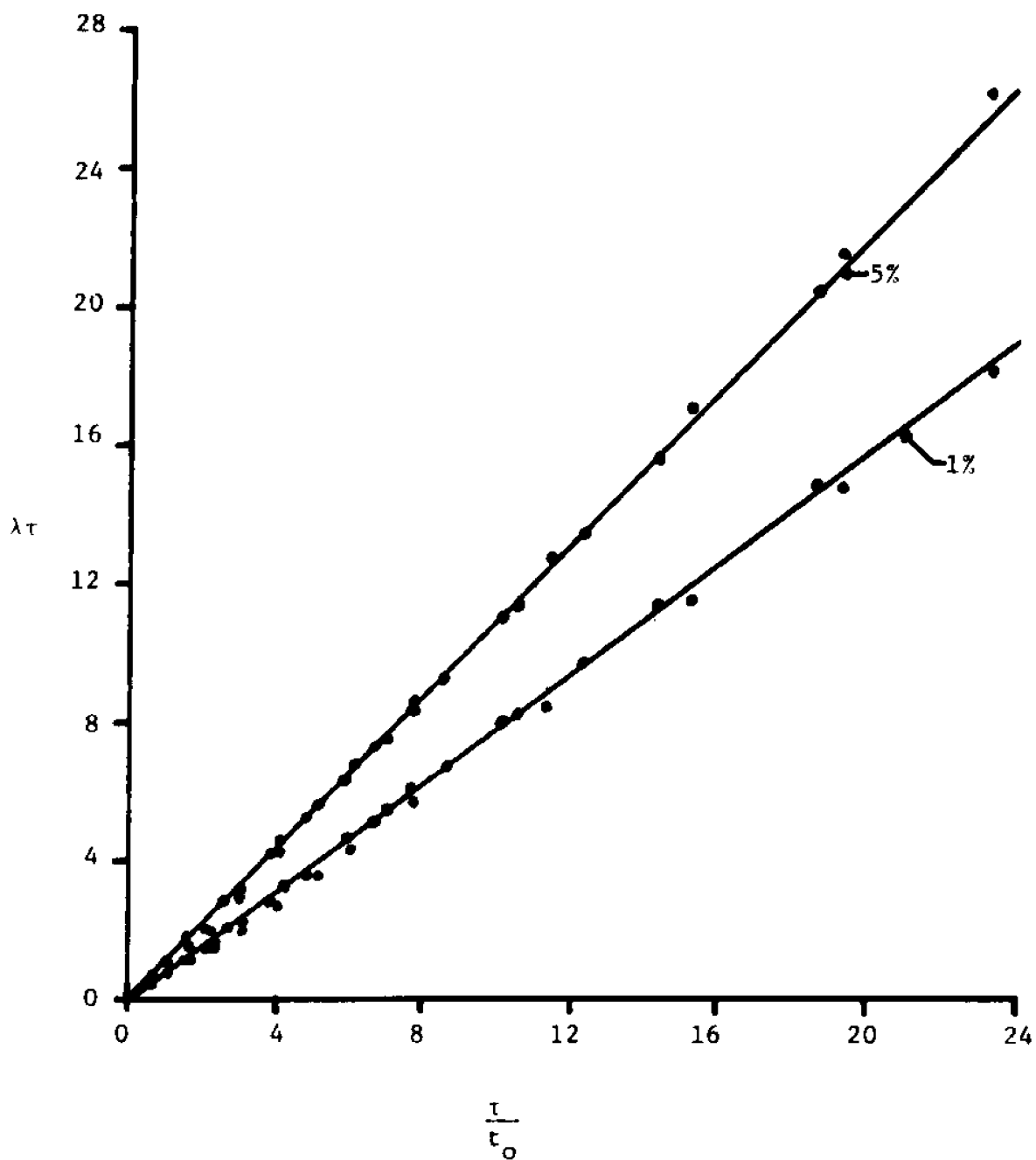
The correlation of the tuning parameter to the parameters of the first-order-lag-plus-dead-time model of the process is presented in Figure 3-5. The plot is given in terms of the dimensionless quantities $\lambda\tau$ and τ/t_o . The 1 and 5 percent overshoot performance criteria are labeled 1% and 5%, respectively. Each data point represents a trial and error calculation on λ to obtain the desired closed-loop response for a given set of process parameters. This was repeated for different process parameters ranging from critically damped to highly overdamped with the dead-time ranging from short to long. The specific process parameters used are presented in Appendix F. A Runge-Kutta-Simpson integration routine was used to solve the closed-loop equations and is listed in Appendix G.

Although the data are somewhat scattered, a straight line can be drawn through the data points well within the accuracy of determining model parameters from plant data. This straight line implies that λ ,

TABLE 3-2
CONTROLLER SYNTHESIS TUNING EQUATIONS

Controller	Process Model	Controller Parameters		
		Gain KK_c	Reset Time T_i	Preact Time T_d
PI	$\frac{Ke^{-t_o s}}{\tau s + 1}$	$\frac{\lambda \tau}{(1 + \lambda t_o)}$	τ	-
PID	$\frac{Ke^{-t_o s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\lambda(\tau_1 + \tau_2)}{(1 + \lambda t_o)}$	$\tau_1 + \tau_2$	$\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
	$\frac{cKe^{-t_o s}}{s^2 + bs + c}$	$\frac{\lambda b}{c(1 + \lambda t_o)}$	b/c	$1/b$

FIGURE 3-5
TUNING PARAMETER FOR A PI CONTROLLER AS A FUNCTION
OF THE FIRST-ORDER-LAG-PLUS-DEAD-TIME MODEL PARAMETERS



the tuning parameter, is only a function of the dead-time, t_o . The equations for these lines can be determined from the plot and are as follows:

$$\lambda \cdot t_o = 1.10; \quad \text{for 5\% overshoot criteria}$$

and

[3-2]

$$\lambda \cdot t_o = 0.790; \quad \text{for 1\% overshoot criteria}$$

Substituting these equations into the controller gain relationship results in the following:

$$KK_c = 0.524 \frac{\tau}{t_o}; \quad \text{for 5\% overshoot criteria}$$

and

[3-3]

$$KK_c = 0.441 \frac{\tau}{t_o}; \quad \text{for 1\% overshoot criteria}$$

These expressions give the PI controller tuning relationships as a function of the model parameters only. Therefore, the PI controller gain, K_c , can be calculated directly from the first-order-lag-plus-dead-time model parameters or by using the equation in Table 3-2 and Figure 3-5.

Results for PID Controller

When the process reaction curve is fitted with a second-order-lag plus-dead-time model, a proportional-plus-integral-plus-derivative (PID) controller will result. This was shown in Chapter II and the resulting tuning equations for the PID controller are also summarized in Table 3-2. The table shows both the factored and polynomial forms of the second-order model transfer function and tuning relationships. Here, the reset time, T_1 , and the preact time, T_d , are functions of the time constants or damping ratio and natural frequency, and the controller gain is a function of the tuning parameter, λ , and all the model parameters.

FIGURE 3-6

TUNING PARAMETER FOR A PID CONTROLLER AS A FUNCTION OF

THE SECOND-ORDER-LAG-PLUS-DEAD-TIME-MODEL

PARAMETERS FOR 5% OVERSHOOT CRITERIA

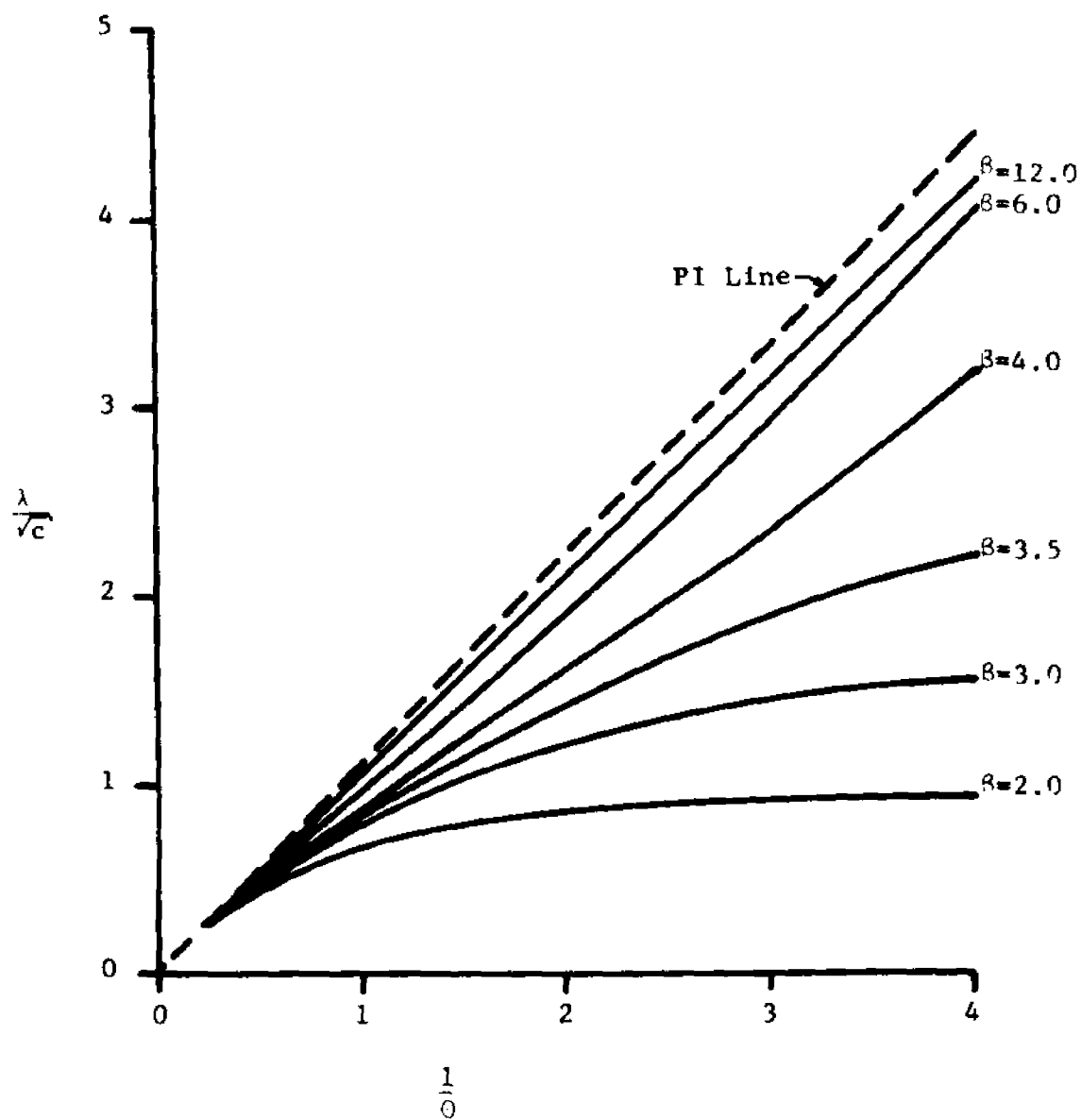
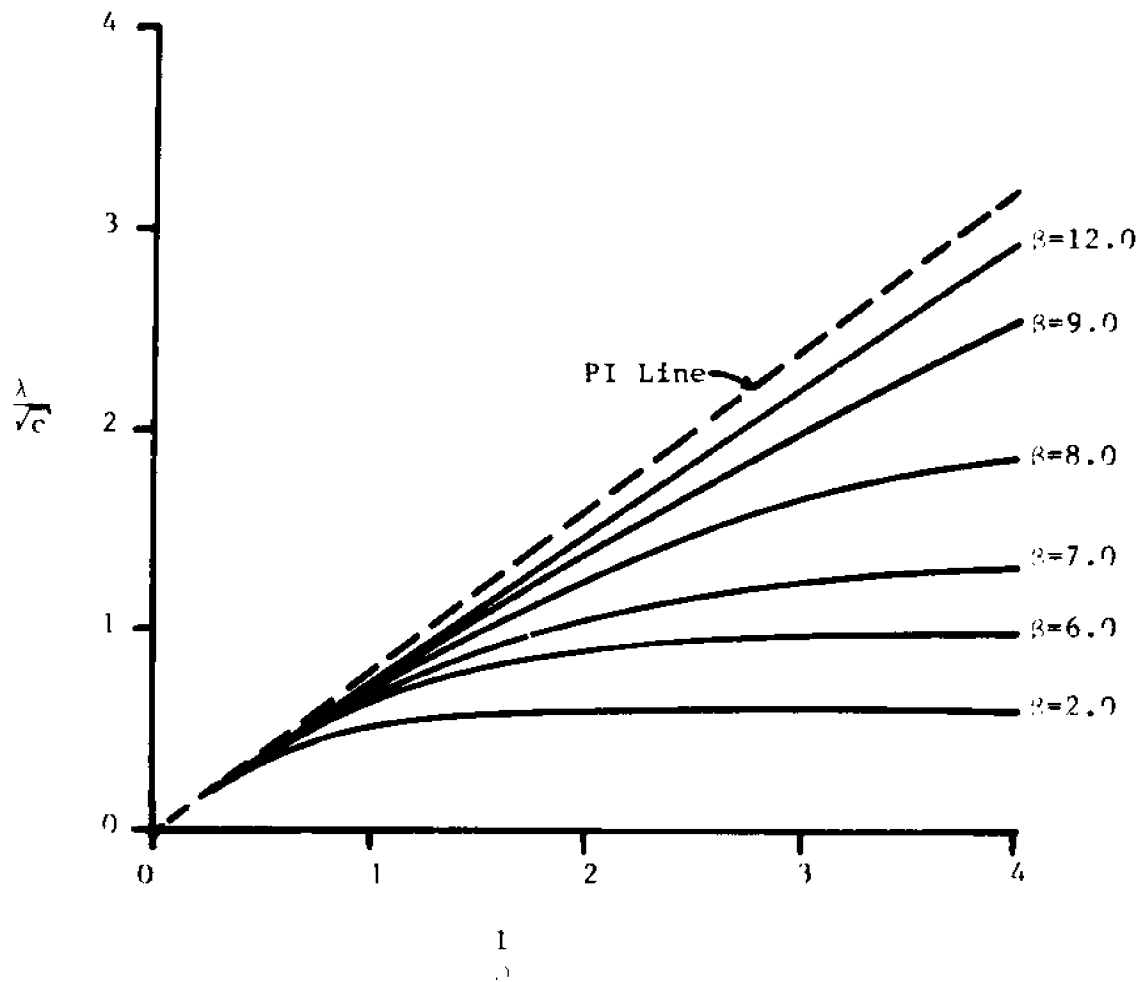


FIGURE 3-7
 TUNING PARAMETER FOR A PID CONTROLLER AS A FUNCTION OF
 THE SECOND-ORDER-LAG-PLUS-DEAD-TIME MODEL
 PARAMETERS FOR 1% OVERSHOOT CRITERIA



The correlations between the tuning parameter and the process model parameters for the 5 and 1 percent overshoot performance criteria are presented in Figures 3-6 and 3-7, respectively. These plots are for PID controller tuning only, and are presented in terms of the following dimensionless parameters.

λ/\sqrt{c} - dimensionless tuning parameter

$\theta = t_o\sqrt{c}$ - dimensionless dead-time

$\beta = b/\sqrt{c}$ - damping parameter ($\zeta = 1/2 b/\sqrt{c}$)

The curves were generated by repetitive trial and error calculations using the secant method on the tuning parameter to obtain the desired response for a given set of process parameters. The process parameters used to obtain the data points are presented in Appendix F.

Figures 3-6 and 3-7 show that as the damping ratio increases, the curve will approach the straight line produced in Figure 3-5 for the tuning of the PI controller based on the first-order-lag-plus-dead-time model. These straight lines are illustrated on Figures 3-6 and 3-7 and are labeled PI line. These figures show that, near the critically damped region, as the dead-time decreases the process gain becomes less dependent on it and becomes a function only of the damping parameter. However, as the damping ratio increases and the process approaches a first-order response, the gain becomes again a function of the dead-time, approaching the first-order (PI) line. This suggests a competition between the effects of the dead-time and the smallest time-constant on the closed-loop response.

The critically damped short dead-time system deviates most from the straight line produced by the first-order model tuning. This is what would be expected since the first-order-lag-plus-dead-time model

does not provide a good fit for the critically damped case. Also at high damping ratios, as the dimensionless dead-time becomes small ($1/\theta > 1.0$) it requires a larger gain to make the system overshoot. Inspection of the two figures also shows that it is easier to achieve the 1% overshoot than the 5% overshoot for small dead-time.

Now the PID controller gain, K_c , can be calculated by using the equations in Table 3-2 and Figure 3-6 or Figure 3-7 and the parameters of a second-order-lag-plus-dead-time model at the process.

In summary, a PI controller can be tuned by fitting a first-order-lag-plus-dead-time model to the process reaction curve to determine the model parameters. Knowing these parameters, Figure 3-5 can be used to determine λ and then the PI tuning equation in Table 3-2 can be used to calculate the settings. Or omit the need for determining λ , by using equation 3-3 to determine the controller gain. A PID controller can be tuned by fitting a second-order-lag-plus-dead-time model to the process reaction curve to determine the model parameters. Knowing these parameters, use Figure 3-6 or 3-7 to determine λ and then the PID tuning equation in Table 3-2 can be used to determine the controller settings.

Discussion

It should be noted from Table 3-2 that the controller gain increases as the tuning parameter increases but the gain asymptotically reaches a limit as the value of the tuning parameter becomes very large. The maximum value of the gain can be found by taking the limit of the gain formulas as the tuning parameter approaches infinity. This is illustrated by the following equations:

PI Controller

$$\lim_{\lambda \rightarrow \infty} KK_c = \lim_{\lambda \rightarrow \infty} \frac{\lambda \tau}{1+t_o \lambda} = \frac{\tau}{t_o} \quad [3.4]$$

PID Controller

$$\lim_{\lambda \rightarrow \infty} KK_c = \lim_{\lambda \rightarrow \infty} \frac{\lambda b}{c(1+t_o \lambda)} = \frac{b}{ct_o}$$

or

[3.5]

$$\lim_{\lambda \rightarrow \infty} KK_c = \lim_{\lambda \rightarrow \infty} \frac{\lambda(\tau_1 + \tau_2)}{1+t_o \lambda} = \frac{\tau_1 + \tau_2}{t_o}$$

These results show that the maximum value of the controller gain is only a function of the parameters of the process model and not a function of the tuning parameter. This does not imply that the controller gain can not be set larger than the value determined from equation 3.4 or 3.5 to achieve a desired response in the field. However, if this is done, it must be understood that tuning beyond the limits of equation 3.4 or 3.5 is not tuning by varying λ , the tuning parameter, but changing the value of t_o , the process model dead-time, or K , the process model gain.

It should also be noted that if the process model is an exact representation of the process, the only term which could cause overshoot is the dead-time approximation. Under these conditions as the dead-time becomes small and approaches zero the closed-loop system no longer overshoots and the controller gain could theoretically be set to infinity to obtain a zero rise time. However, practical considerations such as valve saturation and wear would impose a limit on the value of the gain. Also, under normal conditions, process nonlinearities and model inaccuracies would inhibit perfect compensation of process time lags.

The beauty of the synthesis technique is that it allows the control engineer to back off on the controller gain to prevent valve saturation, etc., without worrying about the other controller parameters.

As stated earlier, the PID controller gain approaches the PI controller gain for highly overdamped processes. This is evident because as the damping ratio increases, the first-order model time constant approaches the dominant second-order model time constant and the smaller second-order model time constant approaches zero. Thus, the PI controller reset time, τ , approaches the PID controller reset time, $\tau_1 + \tau_2$, and the PID controller preact time, $\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$, approaches zero. This indicates that beyond some value of the damping ratio, it is no longer feasible to utilize a PID controller to control the process. From Figure 3-6, a value of b/\sqrt{c} equal to 6.0 seems to be a good choice for the 5% overshoot criteria. This corresponds to a ratio of the second-order time constants of 0.025. From Figure 3-7, a value of b/\sqrt{c} equal to 9.0 seems to be a good choice for the 1% overshoot criteria. This corresponds to a ratio at the second-order time constants of 0.0127. Time constants of this magnitude would be difficult to obtain from a process reaction curve, considering the graphical construction required to obtain the parameters of a second-order model. Therefore, processes of this magnitude of damping ratio should be fitted with a first-order model resulting in a PI controller.

Comparison of Controllers

Comparison of the results of controller synthesis tuning will be made primarily with the results obtained by using Ziegler-Nichols (3) and Rovira (4) tuning formulas. Although these techniques were not developed on the same performance criteria, they are the most widely accepted and utilized in industry and form a good basis for comparison.

As mentioned earlier, Ziegler-Nichols tuning is based on quarter-decay ratio performance and Rovira offers two sets of tuning relationships for set-point changes, one for IAE, integral of the absolute value of the error, and one for ITAE, integral of the time-weighted absolute value of the error. ITAE criteria penalizes the system more for errors which occur late in time and less for errors which occur early in time. IAE criteria is not so critical of small errors which may occur late in time and generally results in less overshoot. Since the controller synthesis tuning relationships developed here are based on % overshoot performance criteria, IAE criteria was chosen for the comparison basis.

Since the three techniques utilize the first-order-lag-plus-dead-time model of the process to tune the parameters of the PI controller, the tuning equations can be compared. These equations are summarized and presented in Table 3-3. From this table it can be shown that the quarter decay ratio gain is 1.75 times that of the 5% overshoot gain and the IAE gain is $1.47 \left(\frac{t_0}{\tau}\right)^{0.14}$ time that of the 5% overshoot gain. The 1% overshoot gain is approximately 80% of the 5% overshoot gain. The reset time, T_i/τ , for both 5% and 1% overshoot is unity, while both quarter-decay ratio and IAE reset times are functions of τ/t_0 . The controller gain and integral

TABLE 3-3
TUNING RELATIONSHIPS

Controller	Ziegler-Nichols (3) (quarter-decay ratio)	Rovira (4) (minimum IAE)	Synthesis	
			5% overshoot	1% overshoot
Proportional - plus - Reset	$KK_c = 0.9 \left[\frac{t_o}{\tau} \right]^{-1.0}$ $\frac{T_i}{\tau} = 3.33 \frac{t_o}{\tau}$	$KK_c = 0.758 \left[\frac{t_o}{\tau} \right]^{-0.861}$ $\frac{T_i}{\tau} = \frac{1}{1.02 - 0.323 t_o/\tau}$	$KK_c = 0.524 \left[\frac{t_o}{\tau} \right]^{-1.0}$ $\frac{T_i}{\tau} = 1.0$	$KK_c = 0.441 \left[\frac{t_o}{\tau} \right]^{-1.0}$
Proportional - plus - Reset - plus - Rate	$KK_c = 1.2 \left[\frac{t_o}{\tau} \right]^{-1.0}$ $\frac{T_i}{\tau} = 2.0 \left[\frac{t_o}{\tau} \right]$ $\frac{T_d}{\tau} = 0.5 \left[\frac{t_o}{\tau} \right]$	$KK_c = 1.086 \left[\frac{t_o}{\tau} \right]^{-0.869}$ $\frac{T_i}{\tau} = \frac{1}{0.74 - 0.13 t_o/\tau}$ $\frac{T_d}{\tau} = 0.348 \left[\frac{t_o}{\tau} \right]^{0.914}$	$KK_c = \frac{\lambda b}{c(1 + \lambda t_o)} = \frac{\lambda(\tau_1 + \tau_2)}{1 + \lambda t_o}$ $T_i \sqrt{c} = \frac{b}{\sqrt{c}} \text{ or } \frac{T_i}{\sqrt{\tau_1 \tau_2}} = \frac{\tau_1 + \tau_2}{\sqrt{\tau_1 \tau_2}}$ $T_d \sqrt{c} = \frac{\sqrt{c}}{b} \text{ or } \frac{T_d}{\sqrt{\tau_1 \tau_2}} = \frac{\sqrt{\tau_1 \tau_2}}{\tau_1 + \tau_2}$	

time for the three techniques are presented in graphical form as a function of the dimensionless model parameter, τ/t_0 , in Figures 3-8 and 3-9, respectively. These figures show that the IAE tuning is very similar to synthesis tuning for 5% overshoot criteria except in the region of small values of the dimensionless first-order model parameter. The IAE gain, KK_c , ranges from -10% to +25% of the 5% overshoot gain. The integral action, τ/T_i , asymptotically approaches the value of the synthesis integral action for values of τ/t_0 greater than 3. Quarter-decay ratio tuning, as expected, has a much higher gain which produces a faster rise time but this faster rise time results in a larger overshoot. The quarter-decay ratio integral time, T_i , is only a function of the model dead-time and produces a straight line in Figure 3-9.

Some typical responses which could be expected by using these tuning relationships are presented in the next four figures. For each figure, the process parameters and the first-order model parameters are presented in Table 3-4 and the PI Controller settings for each technique are presented in Table 3-5. The closed-loop responses for the critically damped process with short and long dead-time are presented in Figures 3-10 and 3-11, respectively. The closed-loop responses for the overdamped process with short and long dead-time are presented in Figure 3-12 and 3-13 respectively. The responses for the different techniques are denoted as follows:

Z-N - Ziegler-Nichols (3) - quarter decay ratio criteria

ROV - Rovira (4) - minimum IAE criteria

5% - Controller Synthesis - 5% overshoot criteria

1% - Controller Synthesis - 1% overshoot criteria

FIGURE 3-8
PI CONTROLLER GAIN AS A FUNCTION OF THE
DIMENSIONLESS FIRST-ORDER MODEL PARAMETER

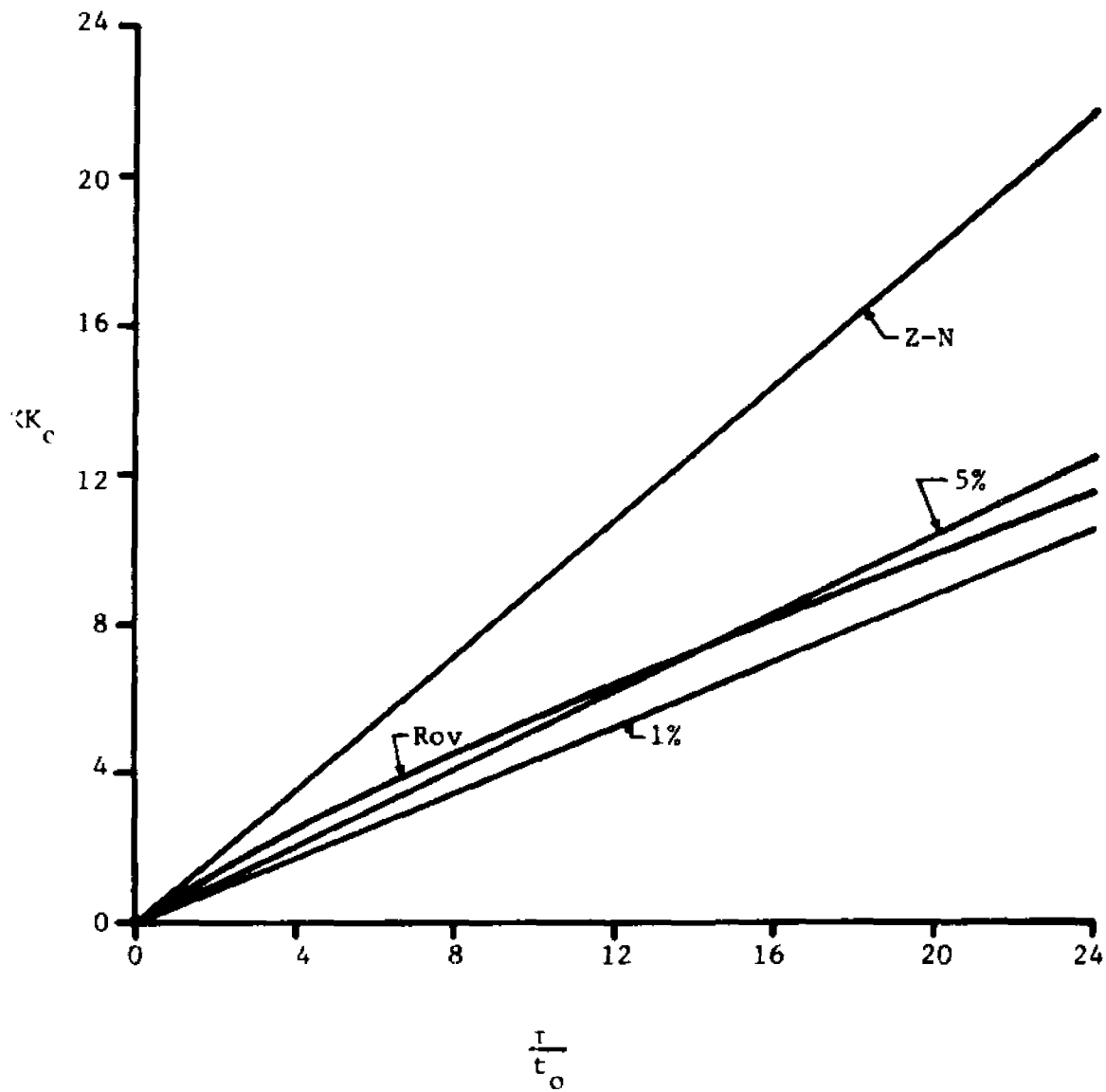


FIGURE 3-9
 PI CONTROLLER INTEGRAL ACTION AS A FUNCTION
 OF THE DIMENSIONLESS FIRST-ORDER MODEL PARAMETER

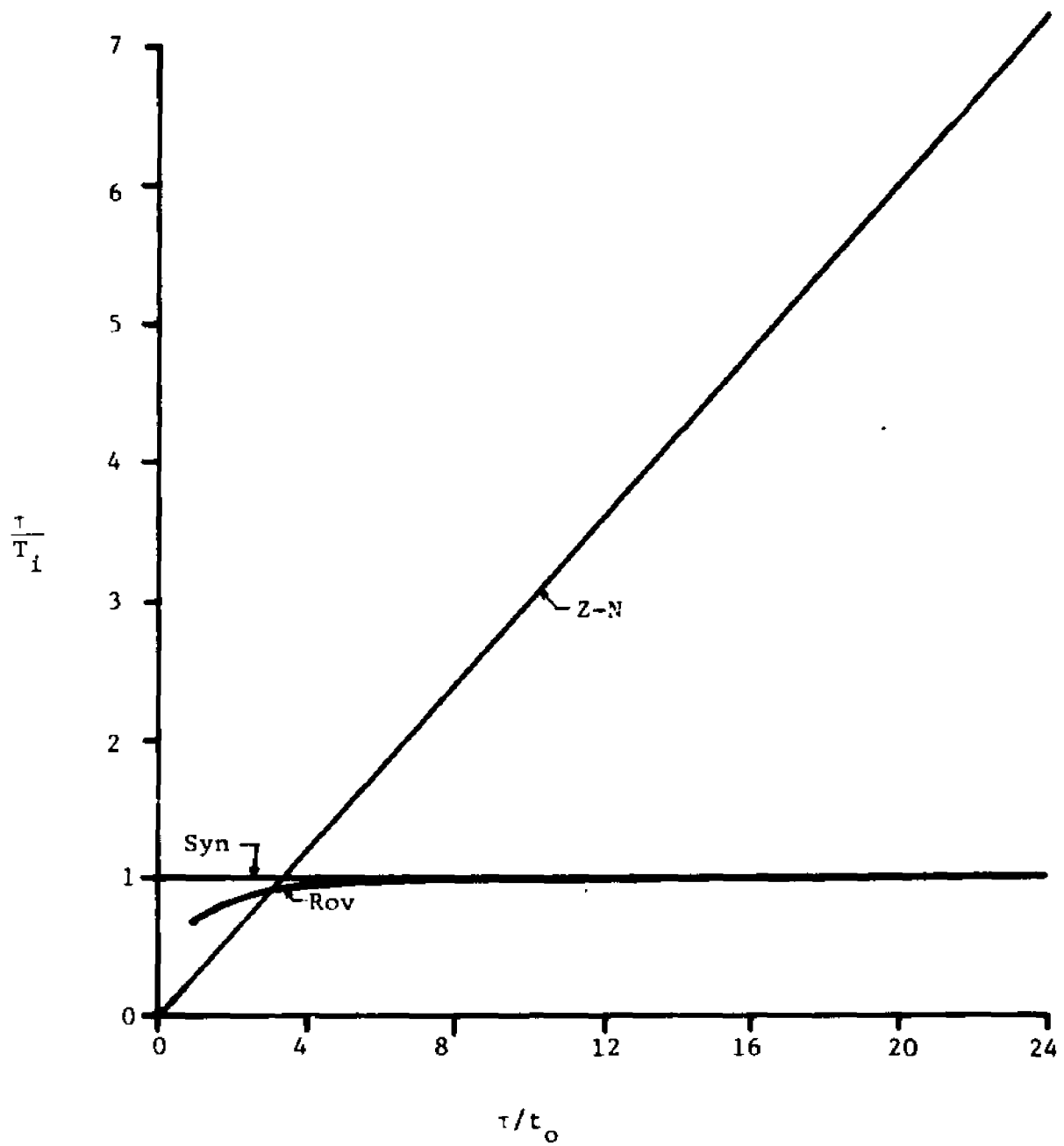


TABLE 3-4
PROCESS AND MODEL PARAMETERS

Figure	Process Parameters				First-order Model Parameters			
	K	$b(\text{min}^{-1})$	$c(\text{min}^{-2})$	$t_o(\text{min})$	K	$\tau(\text{min})$	$t_o(\text{min})$	τ/t_o
3-10	1.0	2.0	1.0	0.25	1.0	1.638	0.758	2.16
3-11	1.0	2.0	1.0	1.0	1.0	1.638	1.508	1.09
3-12	1.0	4.0	1.0	0.25	1.0	3.726	0.531	7.01
3-13	1.0	4.0	1.0	1.0	1.0	3.726	1.281	2.91

TABLE 3-5
PI CONTROLLER SETTINGS

Figure	Ziegler-Nichols (3)		Rovira (4)		Controller-Synthesis		
	Quarter-Decay Ratio		Minimum IAE		5% overshoot	1% overshoot	5% & 1%
	KK_c	T_i (min.)	KK_c	T_i (min.)	KK_c	KK_c	T_i (min.)
3-10	1.945	2.525	1.472	1.883	1.137	0.964	1.639
3-11	0.978	5.025	0.814	2.268	.571	0.485	1.639
3-12	6.311	1.770	4.055	3.831	3.688	3.128	3.731
3-13	2.617	4.274	1.900	4.098	1.530	1.297	3.731

FIGURE 3-10

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY DAMPED PROCESS
WITH SHORT DEAD-TIME FOR A PI CONTROLLER

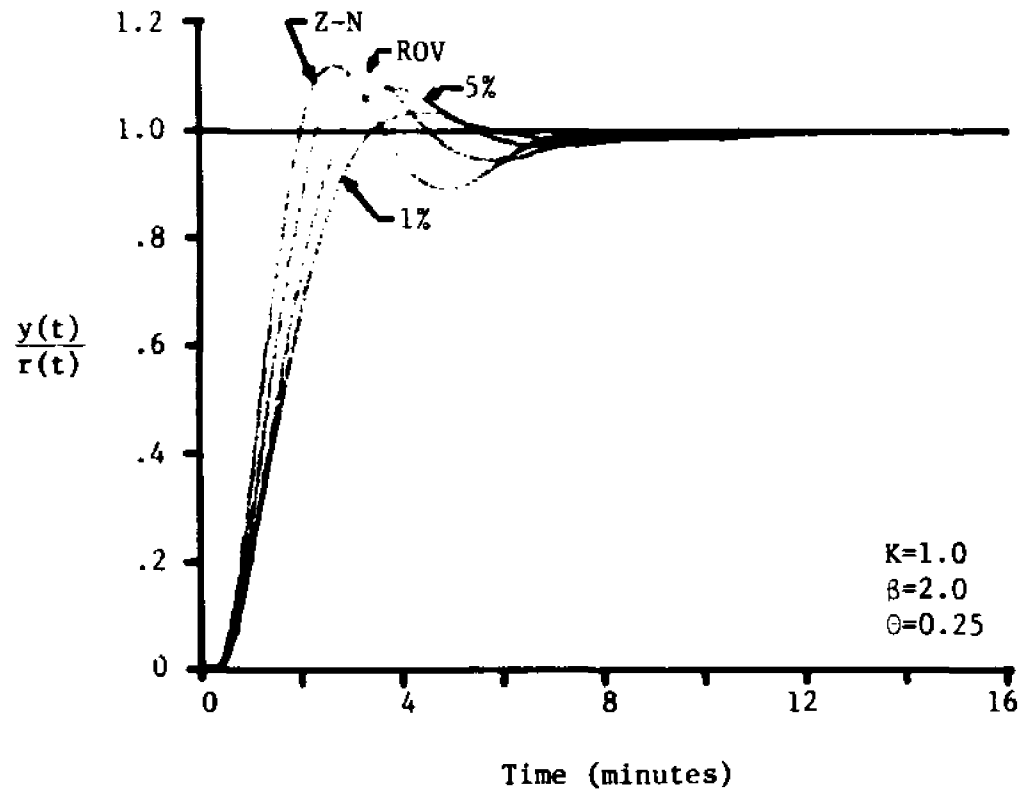


FIGURE 3-11

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY DAMPED PROCESS
WITH LONG DEAD-TIME FOR A PI CONTROLLER

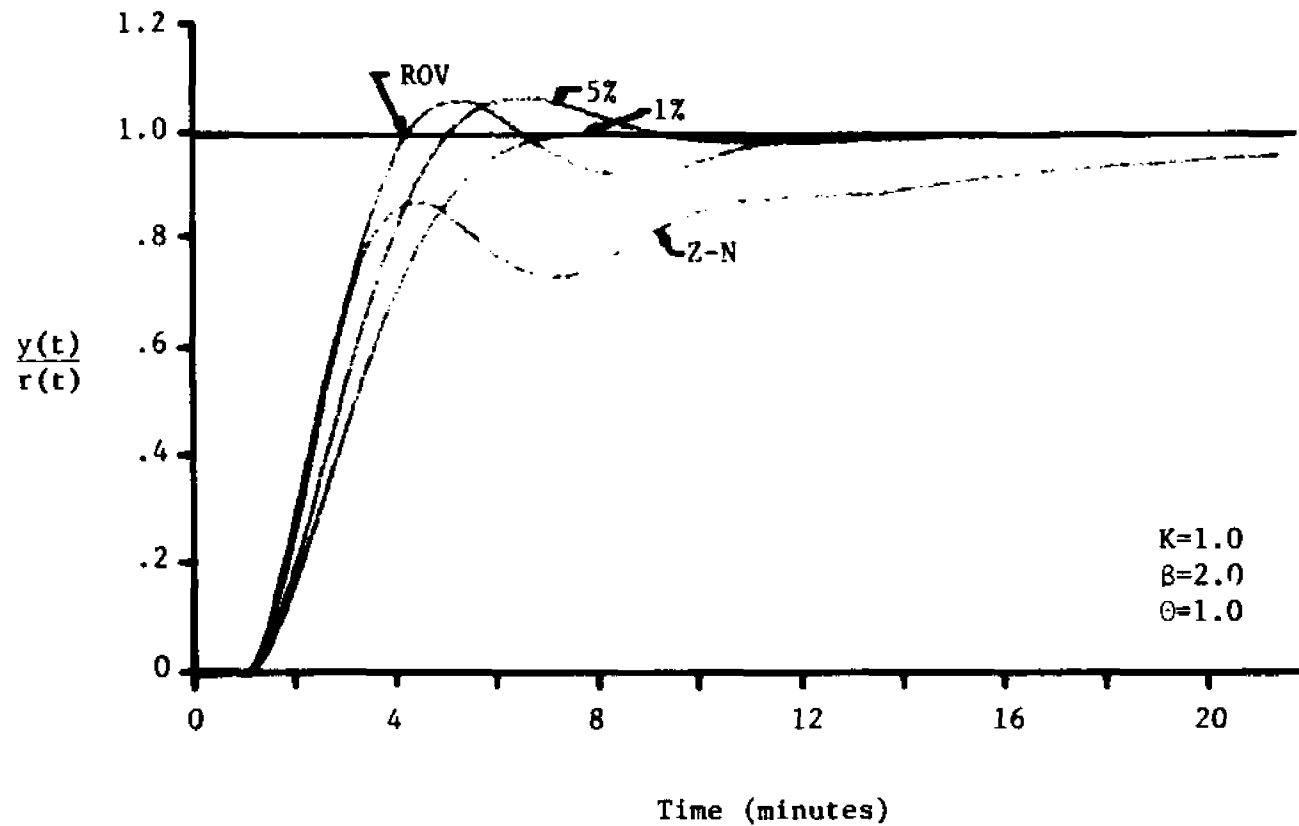


FIGURE 3-12

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED PROCESS
WITH SHORT DEAD-TIME FOR A PI CONTROLLER

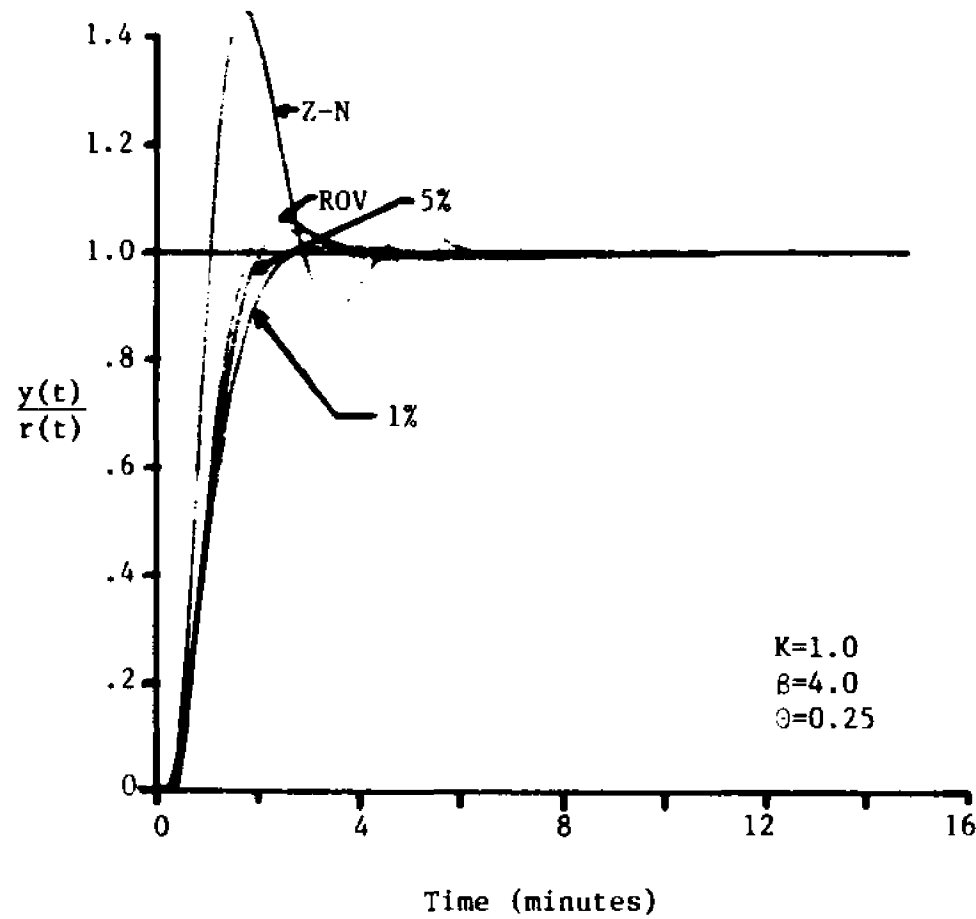
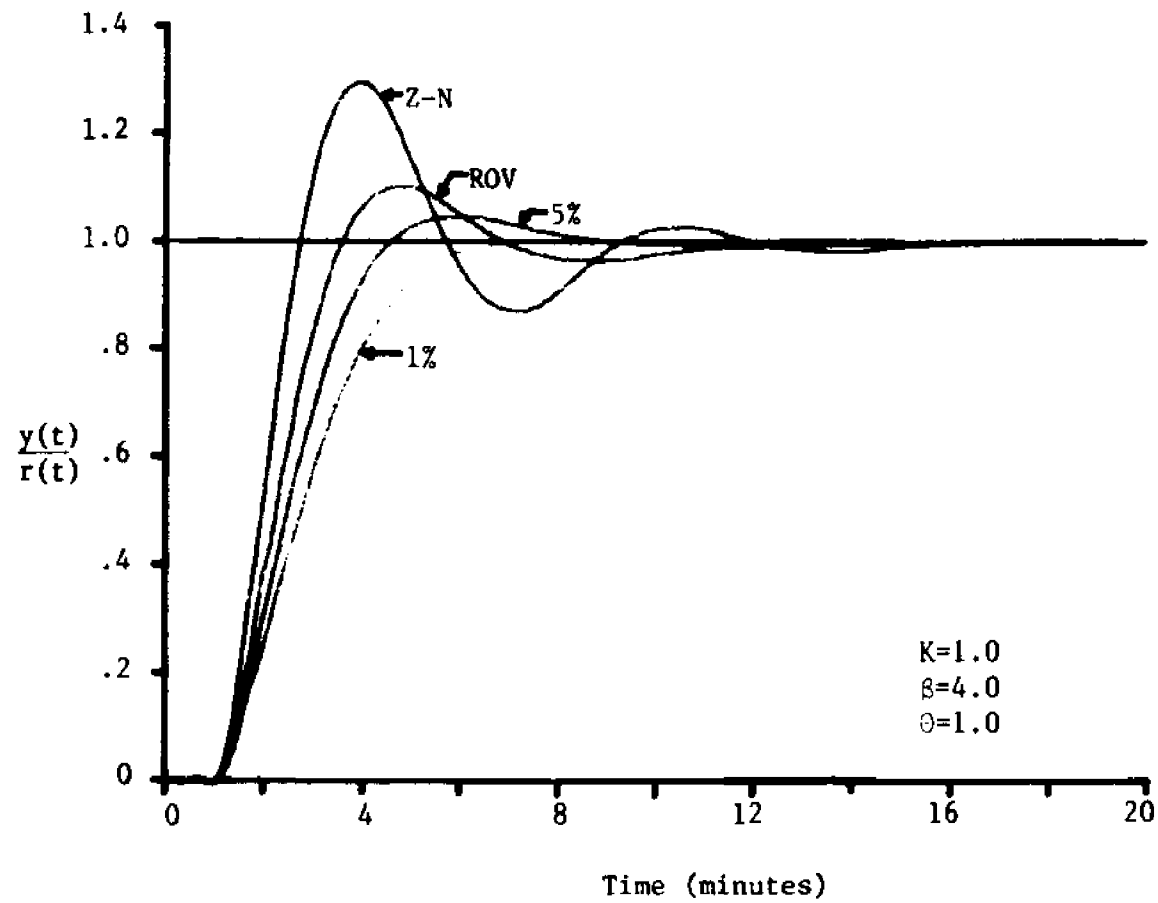


FIGURE 3-13

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED
PROCESS WITH LONG DEAD-TIME FOR A PI CONTROLLER



As mentioned earlier, the straight line approximation for the PI Controller Synthesis settings may not result in the exact overshoot criteria. However, minor adjustment to the controller gain results in the desired response. The controller synthesis gains and overshoots produced by using equation 3.3 for these four figures are presented in Table 3-6. To show the magnitude of the adjustment required to produce the desired overshoot within $\pm 0.01\%$, the correct gains are given in parenthesis in this table.

These closed-loop responses illustrate that even in the cases of larger overshoots, the system dynamics introduced by the controller synthesis technique is not as severe as that introduced by the other techniques. These figures also show that while slightly faster responses are obtainable by IAE and quarter-decay ratio tuning, it is at the expense of more oscillatory system dynamics which is usually undesirable from an industrial standpoint. From this point of view, the controller syntheses tuning is superior because it produces essentially no oscillatory behavior. These results coupled with the fact that the synthesis technique conceptually provides a parameter which can be tuned and the other techniques do not, makes it very adaptable for industrial use.

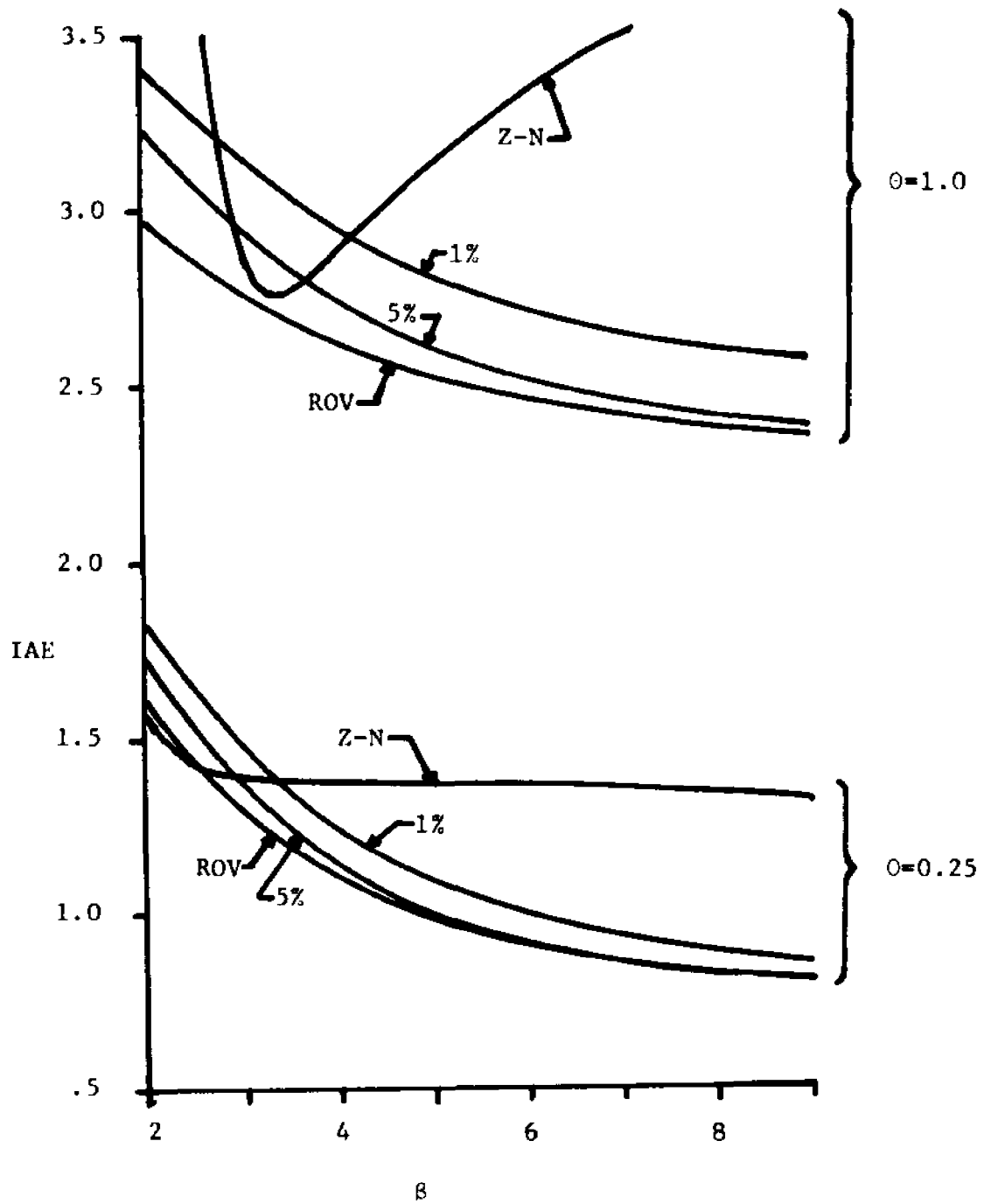
As final comparison of the results obtained by the different tuning techniques for a PI Controller, the IAE criteria will be examined. The integral of the absolute value of the error for each of the techniques are presented as a function of the dimensionless process parameters, β and θ , in Figure 3-14. The controller synthesis results presented here are based on tuning by using equation 3-3. It can be seen from this plot that there is not much

TABLE 3-6
FINE TUNING ADJUSTMENTS
FOR CONTROLLER SYNTHESIS TECHNIQUE

Figure	5% overshoot		1% overshoot	
	KK_c	overshoot	KK_c	overshoot
3-10	1.137 (0.996)	8.7% (5.0%)	0.964 (0.852)	3.9% (1.0%)
3-11	.571 (.547)	6.8% (5.0%)	0.485 (0.488)	0.8% (1.0%)
3-12	3.688 (3.719)	4.8% (5.0%)	3.128 (2.977)	1.6% (1.0%)
3-13	1.530 (1.511)	5.4% (5.0%)	1.297 (1.281)	1.2% (1.0%)

FIGURE 3-14

INTEGRAL OF THE ABSOLUTE VALUE OF THE ERROR FOR A
SECOND-ORDER-PLUS-DEAD-TIME PROCESS WITH A PI CONTROLLER



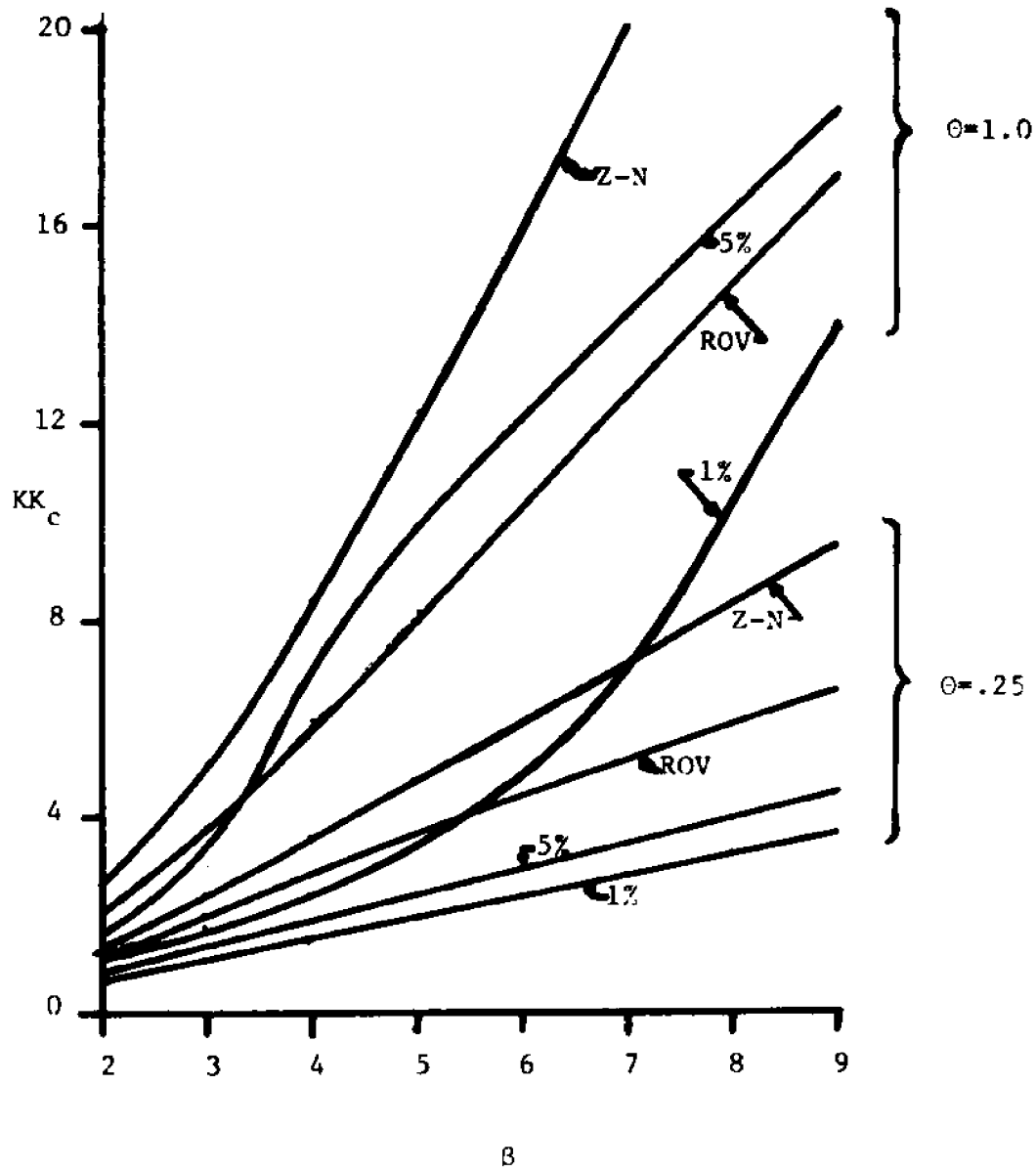
difference between the IAE for IAE tuning and 5% overshoot tuning. The maximum difference is approximately 10%. The IAE of the 1% overshoot tuning varies from 7% to 10% higher than the IAE of the 5% overshoot tuning.

It is more difficult to make a comparison of the PID controllers relationships because the relationships are not based on the same process model. Ziegler-Nichols (3), quarter decay ratio criteria, and Rovira (4), minimum IAE criteria, relationships are based on the first-order-lag-plus-dead-time model of the process and the controller synthesis relationships are based on the second-order-lag-plus-dead-time model of the process. This results in the controller synthesis relationship being a function of the three process model parameters K , τ_1 , τ_2 and t_o or K , b , c , and t_o and the other relationships being a function of K , τ and t_o only. It should be noted here that t_o , the first-order-model dead-time, is equal to t_o , the second-order transfer function dead-time, plus Δ , the effective dead-time introduced by the second-order-lag transfer function. This was discussed earlier and Figures 3-3 and 3-4 can be used to relate these parameters. These figures were used to obtain the PID parameters for Rovira and Zeigler-Nichols techniques.

The PID controller relationships for the three techniques are summarized in Table 3-3. As the equations can not be readily compared, the next three figures provide a graphical means of comparing the tuning parameters.

The controller gains are presented in Figures 3-15 as a function of the process parameter, β , for short dead-time ($\theta = 0.25$) and long

FIGURE 3-15
 PID CONTROLLER GAINS AS A FUNCTION OF THE
 PARAMETERS OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME PROCESS



dead-time ($\theta = 1.0$). The plot indicates that the controller gains are strong functions of both the process dead-time and the process damping parameter. For small values of β , the gain becomes less dependent on the dead-time than at large values of β . However, the plot shows that only the 1% overshoot criteria gain is a moderate function of the dead-time for small β . Quarter-decay ratio has the largest gain for both long and short dead-time and the 1% overshoot criteria gain is smallest for both cases.

The dimensionless reset times are presented in Figure 3-16 as a function of the process parameters. The controller synthesis reset time is independent of the process dead-time. The figure shows that the IAE reset time is only moderately a function of the process dead-time while the quarter-decay ratio reset time is a strong function of the process dead-time. It should be noted that the controller synthesis dimensionless reset time is shown as $T_1\sqrt{c}/\beta$. This was done because the dimensionless time constant $\tau\sqrt{c}$ approaches β as β increases. This is shown in Figure 3-18. Therefore the dimensionless reset time $T_1\sqrt{c}/\beta$ is comparable with $T_1\sqrt{c}/\tau\sqrt{c}$ or T_1/τ . The controller synthesis dimensionless reset time is unity and in general is larger than the quarter-decay ratio dimensionless reset time but smaller than the IAE dimensionless reset time.

The dimensionless rate or preact times are presented in Figure 3-17 as a function of the process parameters. The controller synthesis dimensionless preact time is presented as $T_d\sqrt{c}/\beta$ based on the discussion of Figure 3-18. As can be seen, from Figure 3-17, the quarter-decay ratio and IAE preact times are both strong function of the

FIGURE 3-16

PID RESET TIME IS A FUNCTION OF THE PARAMETERS
OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME PROCESS

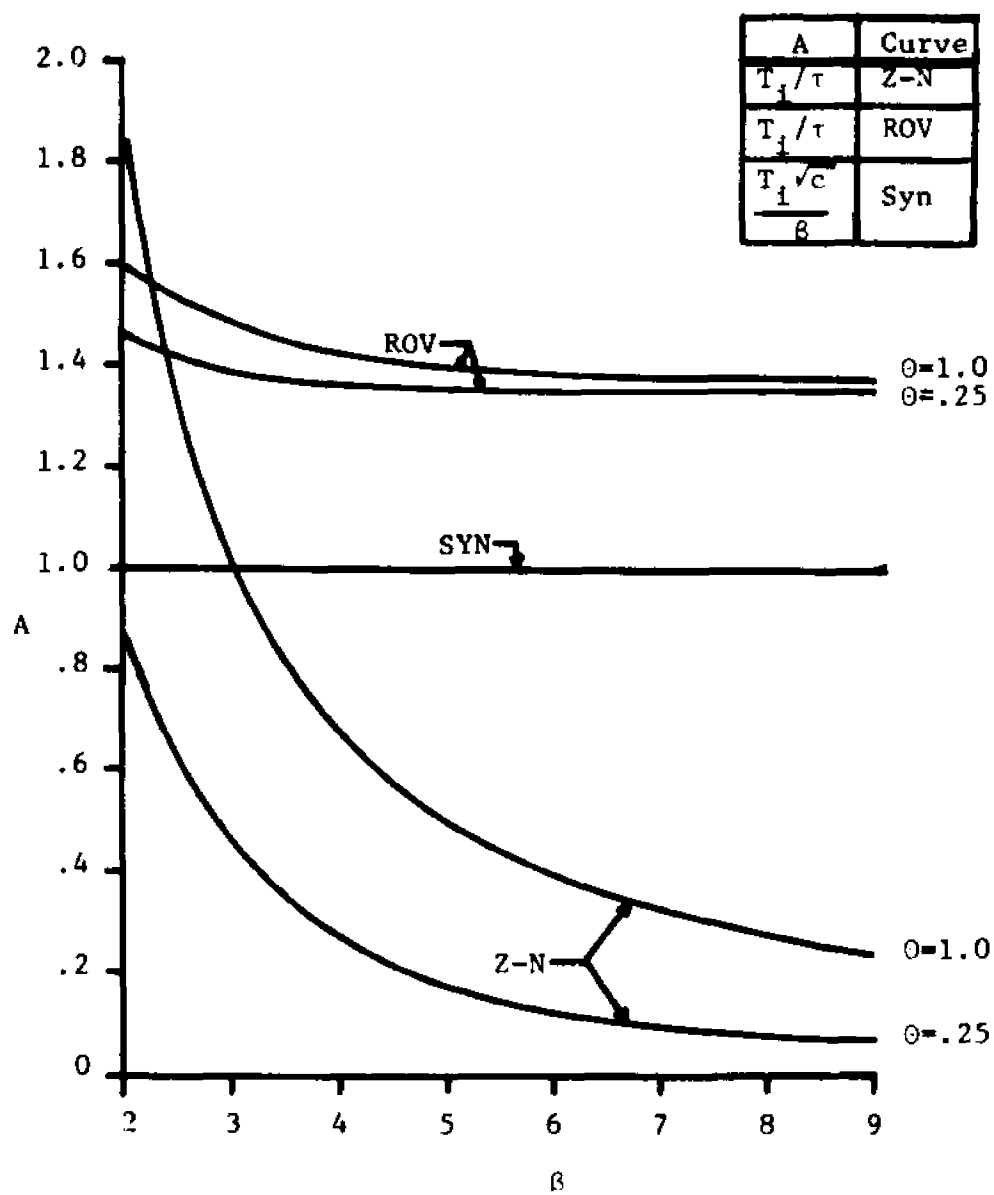


FIGURE 3-17
 PID PREACT TIME AS A FUNCTION OF THE PARAMETERS
 OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME PROCESS

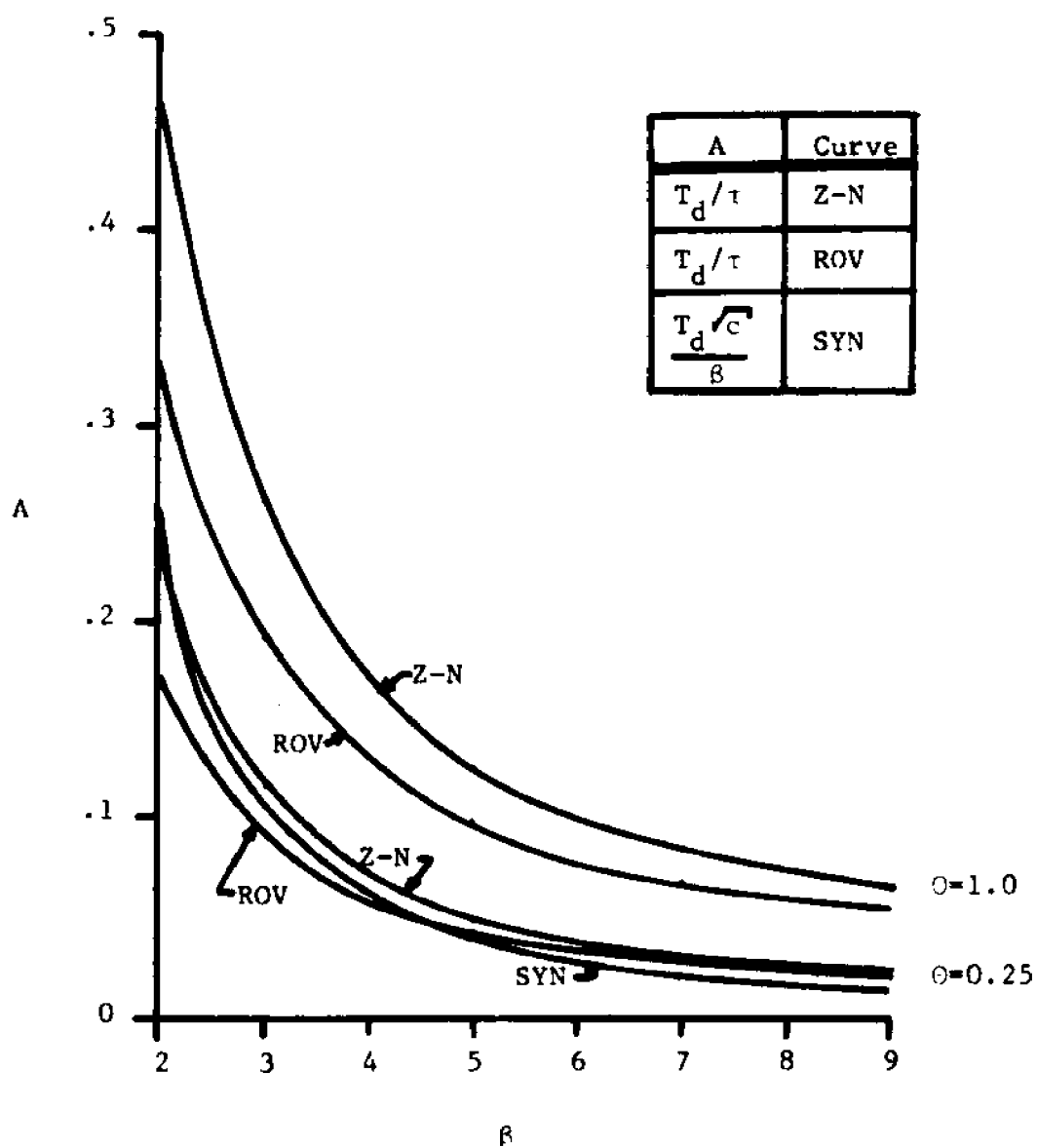
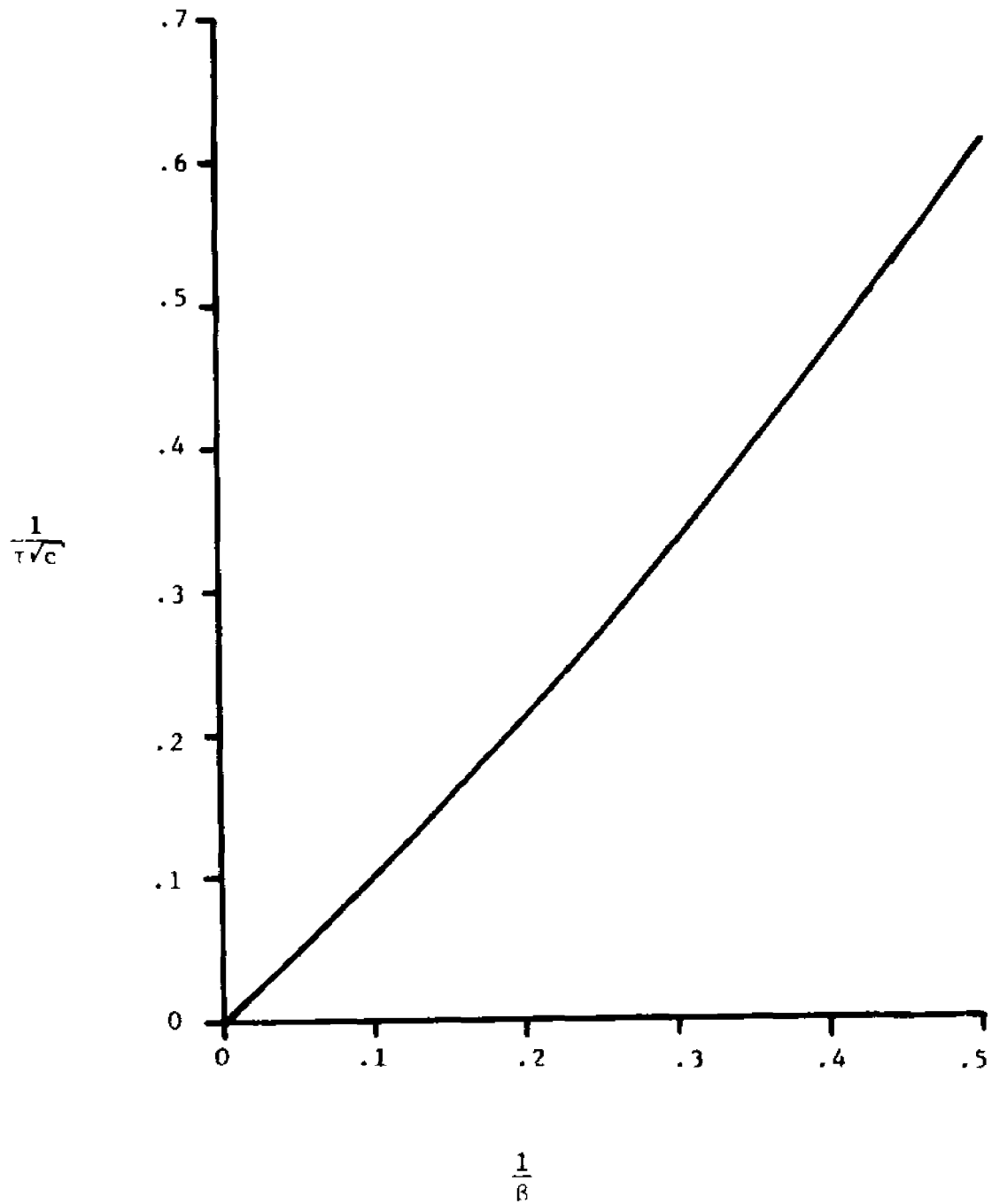


FIGURE 3-18
DIMENSIONLESS FIRST-ORDER MODEL TIME CONSTANT AS A
FUNCTION OF THE SECOND-ORDER MODEL DAMPING PARAMETER



process dead-time. Both techniques specify more derivative action for increasing dead-time. Controller synthesis preact time is not a function of the process dead-time and in general specifies less preact time than the other techniques.

To compare the closed-loop responses of these tuning techniques the next four figures presents some typical responses which could be expected by using these tuning relationships. The process parameters and the parameters of the first-order-lag-dead-time model of the process are presented in Table 3-7 for each of the figures. The PID controller settings are presented in Table 3-8 for the respective tuning techniques for each of the figures. The critically damped process with short and long dead-time are presented in Figure 3-19 and 3-20, respectively, and the overdamped process with short and long dead-time are presented in Figures 3-21 and 3-22, respectively. The different techniques will be labeled as before by the symbols used in the PI controller comparison.

As was the case with the PI controllers, it can be seen from these figures that the controller synthesis tuning introduces less oscillatory behavior in the closed-loop response of the system. Again the plots show that while slightly faster responses are obtainable by IAE tuning and quarter-decay ratio tuning, it is usually at the expense of larger overshoots.

As a final comparison of the results obtained by the different tuning techniques, the IAE or integral of the absolute value of the error will be examined. The IAE of the different techniques are presented in Figure 3-23 as a function of the dimensionless process parameters, β and θ . From this plot, it can be seen that the IAE

TABLE 3-7
PROCESS AND MODEL PARAMETERS

Figure	Process Parameters				First-order Model Parameters			
	K	$b(\text{min.}^{-1})$	$c(\text{min.}^{-2})$	$t_o(\text{min.})$	K	$\tau(\text{min.})$	$t_o(\text{min.})$	τ/t_o
3-19	1.0	2.0	1.0	0.25	1.0	1.638	0.758	2.16
3-20	1.0	2.0	1.0	1.0	1.0	1.638	1.508	1.09
3-21	1.0	4.0	1.0	0.25	1.0	3.726	0.531	7.01
3-22	1.0	4.0	1.0	1.0	1.0	3.726	1.281	2.91

TABLE 3-8
PID CONTROLLER SETTINGS

Figure	Ziegler-Nichols (3)			Rovira (4)			Controller Synthesis			
	Quarter-decay Ratio			Minimum IAE			5%	1%	5% & 1%	
	KK_c	T_i (min.)	T_d (min.)	KK_c	T_i (min.)	T_d (min.)	KK_c	KK_c	T_i (min.)	T_d (min.)
3-19	2.594	1.515	0.378	2.122	2.410	0.282	1.510	1.071	2.0	0.5
3-20	1.304	3.012	0.753	1.167	2.638	0.529	0.800	0.678	2.0	0.5
3-21	8.415	1.062	0.265	5.901	5.155	0.220	4.080	2.422	4.0	0.25
3-22	3.490	2.564	0.641	2.746	5.348	0.488	1.844	1.426	4.0	0.25

FIGURE 3-19

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY DAMPED
PROCESS WITH SHORT DEAD TIME FOR A PID CONTROLLER

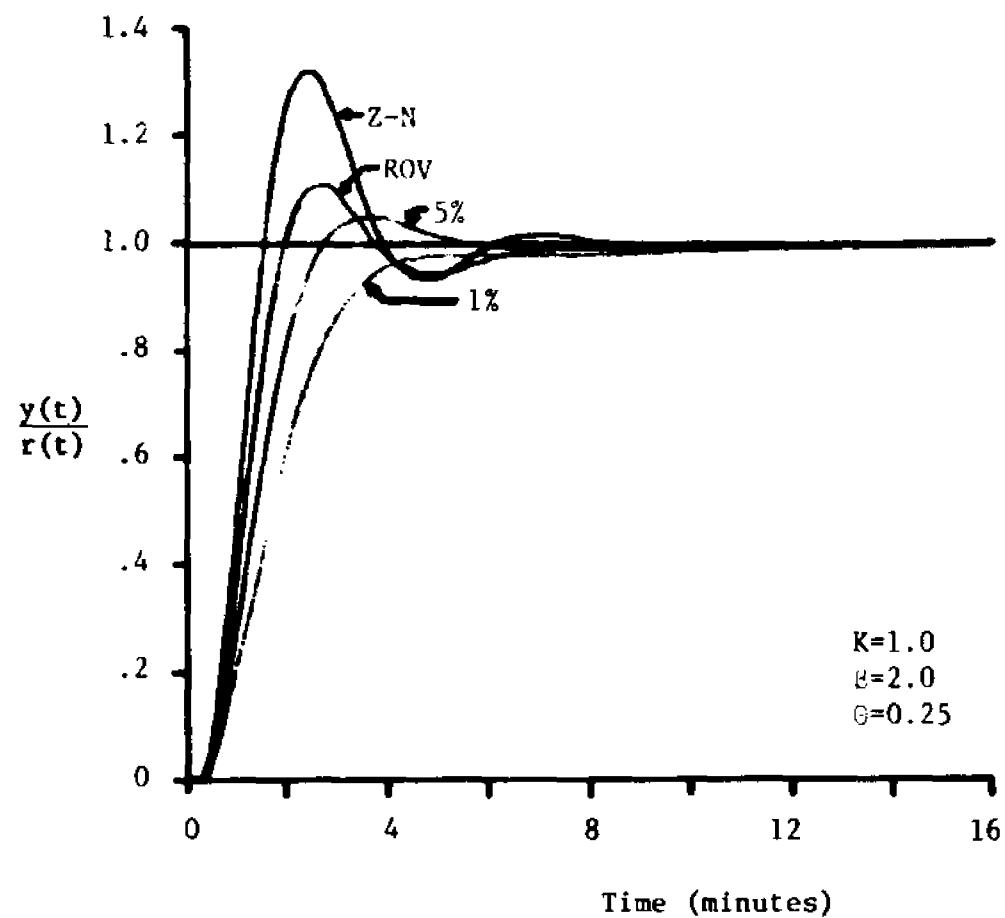


FIGURE 3-20

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY
DAMPED PROCESS WITH LONG DEAD-TIME FOR A PID CONTROLLER

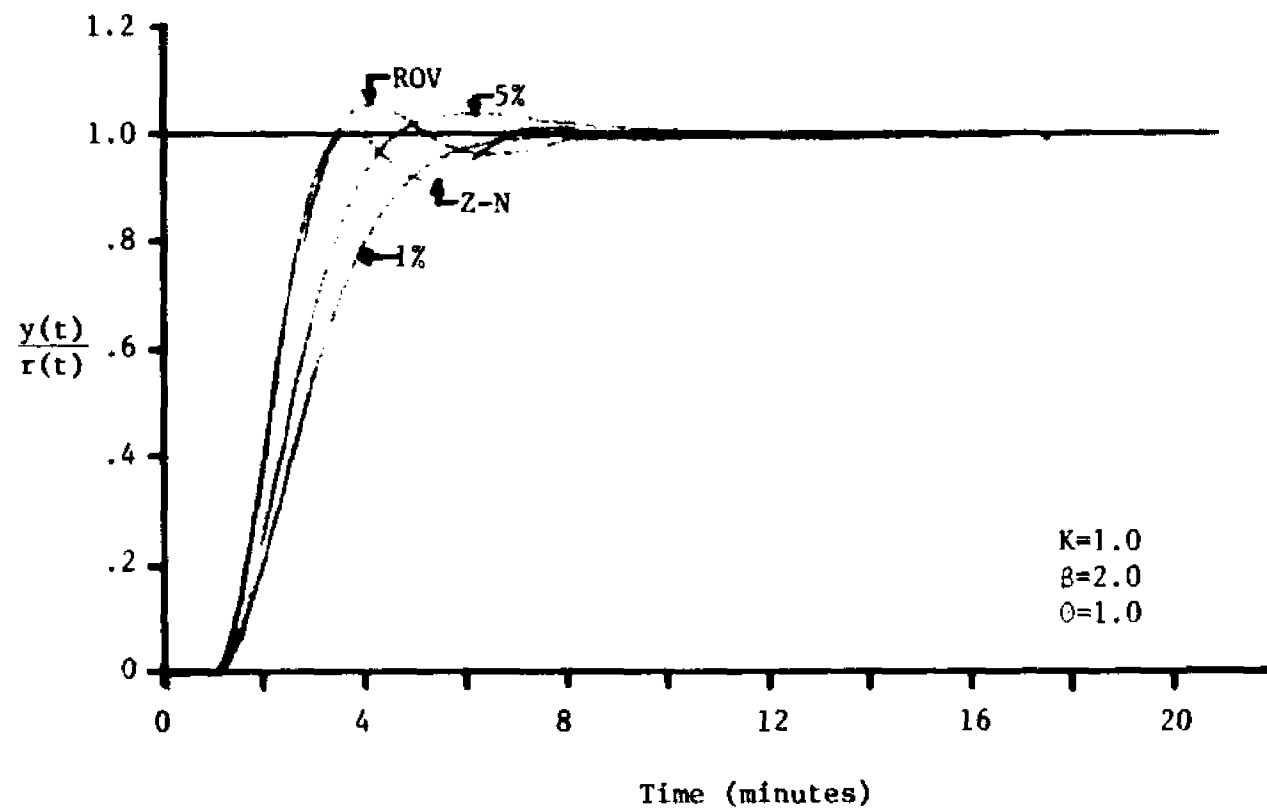


FIGURE 3-21

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED
PROCESS WITH SHORT DEAD-TIME FOR A PID CONTROLLER

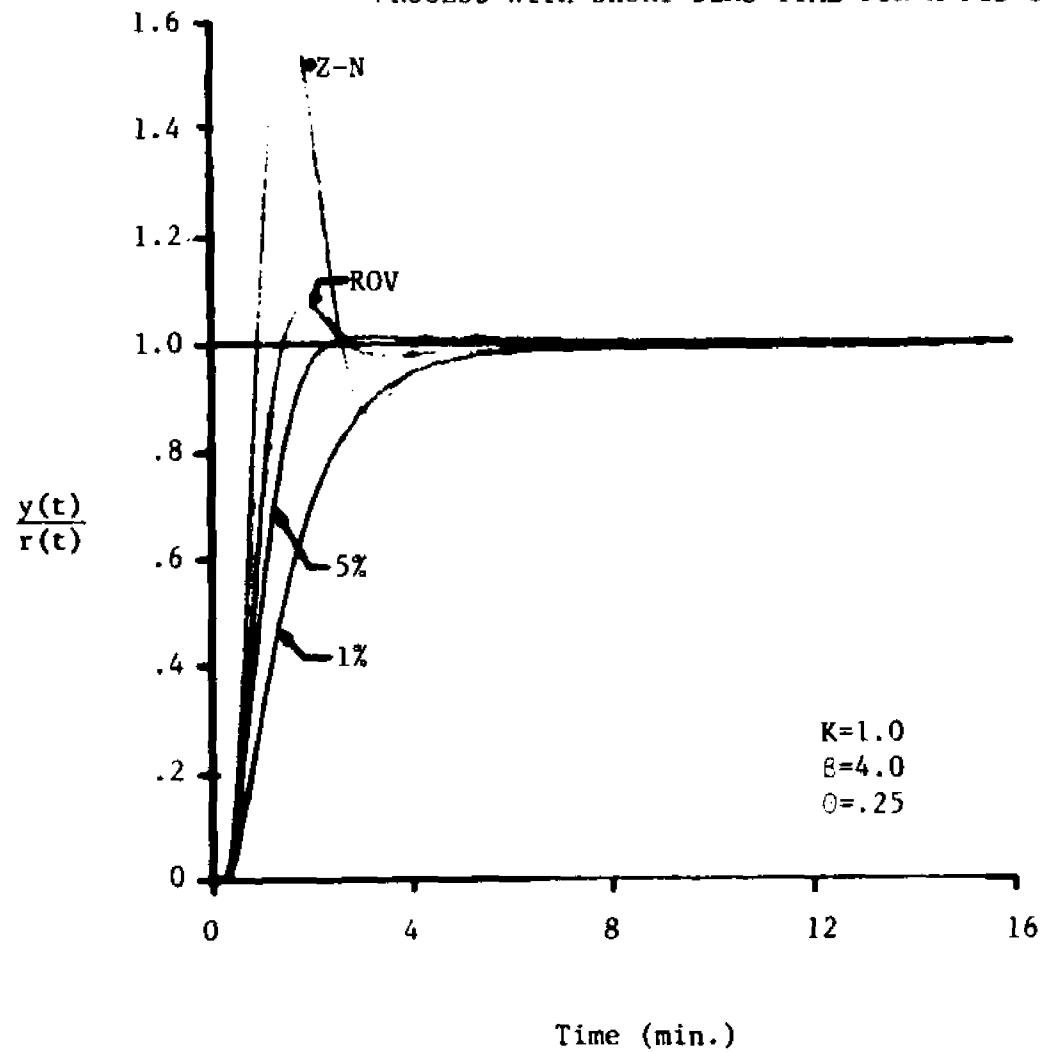


FIGURE 3-22

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED
PROCESS WITH LONG DEAD-TIME FOR A PID CONTROLLER

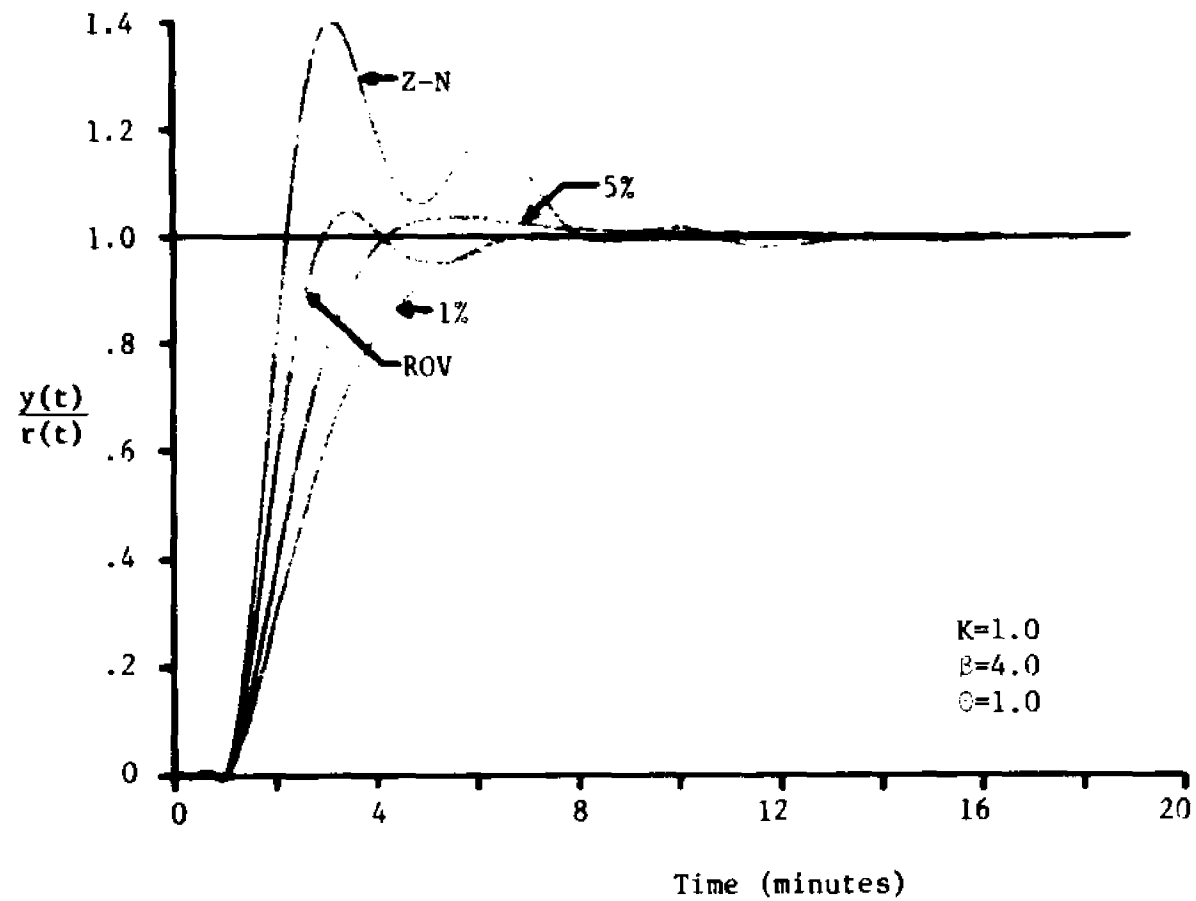
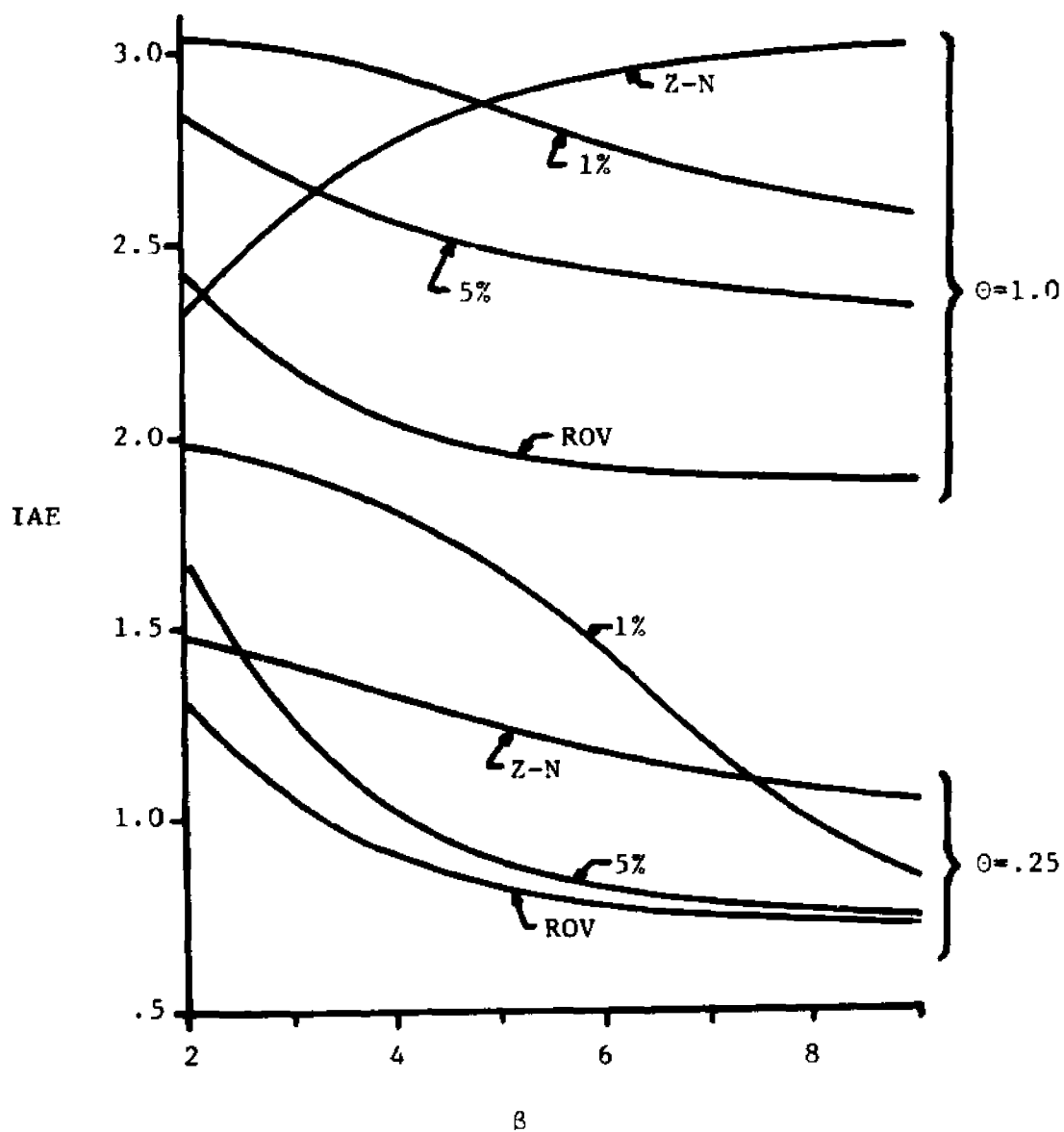


FIGURE 3-23

INTEGRAL OF THE ABSOLUTE VALUE OF THE ERROR FOR A
SECOND-ORDER-LAG-PLUS-DEAD-TIME PROCESS WITH A PID CONTROLLER



for the 5% overshoot tuning and minimum IAE tuning are similar for short dead-time systems but the difference increases with increasing dead-time. However, the maximum difference produced by the 5% overshoot tuning is approximately 20%. The IAE for the 1% overshoot tuning is largest for most cases considered. It is interesting to note that quarter decay ratio produces values of IAE which are sometimes less than the other techniques. In particular, it is minimum for the case where $\beta = 2.0$ and $\theta = 1.0$, but examination of the response for this case (Figure 3-20) reveals that this type of response is very undesirable.

Summary

In this chapter, controller tuning relationships for the synthesis technique were developed and the results were compared with Ziegler-Nichols (3) quarter-decay ratio tuning and Rovira (4) minimum integral of the absolute value of the error tuning. Both the PI and PID controllers were examined in detail.

As the results show, synthesis tuning for the overshoot criteria results in a response which was slightly less oscillatory than the other techniques. However, it should be noted that the synthesis tuning and the IAE tuning were very similar for some of the cases examined. This was especially true for the PI controller for large values of τ/t_0 . A similarity for the PID controller was that the IAE reset time was a very moderate function of the process dead-time.

It is important to note that the reset time for a PI controller or reset and preact times for a PID controller are only a function of the model time constants for the synthesis technique but a

function of the model time constant and dead-time for the other techniques.

In general, the controller synthesis technique offers the control engineer simplicity and flexibility of tuning for a desired response which has not been available to him in the past.

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CHAPTER IV

RESULTS FOR THE OPTIMAL LINEAR REGULATOR TECHNIQUE

Introduction

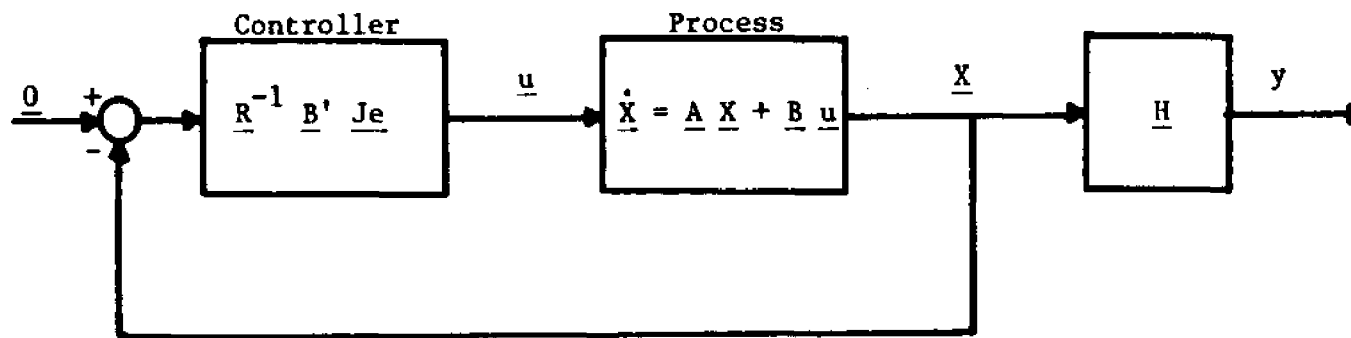
The application of optimal linear regulator theory is another technique which offers some flexibility in obtaining a desired response. The linear regulator is based on the minimization of a quadratic performance or cost function subject to a linear state variable representation of the process. This theory results in a linear feedback control policy which is proportional to the states, thus the name linear state regulator. If the system is time-invariant and certain additional restrictions are imposed on the cost function and the process to guarantee the existence of a minimum, the resulting control policy, illustrated in Figure 4-1, is called the output regulator. This figure is for the single-input single-output time invariant system. It was shown in Chapter II that the systems under consideration meet all restrictions specified by the output regulator.

The performance or cost function of the single-input single-output system is

$$I = \int_0^{\infty} (y^2 + p \dot{u}^2) d\sigma$$

and contains the parameter p which must be specified by the user ($p > 0$) and penalizes the system for excessive movement of the valve. This parameter can be utilized as a tuning parameter since

FIGURE 4-1
OUTPUT REGULATOR BLOCK DIAGRAM



different values of p result in different closed-loop responses. Large values of p result in a sluggish response and as p is decreased the system will respond faster. This is illustrated in Figure 4-2 which shows the closed-loop responses for several values of p for a system consisting of a second-order-lag-plus-dead-time process and a PID controller. The responses are for a unit step-change in set-point. The process parameters are as follows:

$$K \equiv \text{process gain} = 1.0$$

$$b \equiv \text{damping parameter} = 4.0 \text{ time}^{-1}$$

$$c \equiv \text{frequency parameter} = 1.0 \text{ time}^{-2}$$

$$t_o \equiv \text{process dead-time} = 0.5 \text{ time}$$

The controller settings corresponding to the values of p shown on the plot are given in Table 4-1 and are based on a first-order-lag-plus-dead-time model of the process. From Figure 4-2, it can be seen that the faster response obtained by decreasing p beyond some point is at the expense of large overshoot. The large overshoots are accompanied by excessive valve movement as shown in Figure 4-3 which presents the controller output or manipulated variable as a function of time for the closed-loop responses obtained in Figure 4-2. This indicates that a performance criteria must be selected, then the tuning parameter, p , can be correlated to the parameters of the process. However, since the relationships between the controller parameters and the tuning parameter are not simple, the controller settings resulting from the application of the optimal linear regulator will also be correlated to the parameters of the process models.

The performance criteria will be the five percent overshoot

TABLE 4-1
 PID CONTROLLER SETTING BASED ON A
 FIRST-ORDER-LAG-PLUS-DEAD-TIME MODEL

P (min) ²	K_c	T_i (min.)	T_d (min.)
.1	3.975	1.876	0.309
.5	2.515	2.278	0.324
2.0	1.612	2.670	0.333
4.0	1.269	2.871	0.338
10.0	0.906	3.125	0.341

FIGURE 4-2

CLOSED-LOOP RESPONSES OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME

PROCESS WITH A PID CONTROLLER AS A FUNCTION OF THE PENALTY FUNCTION

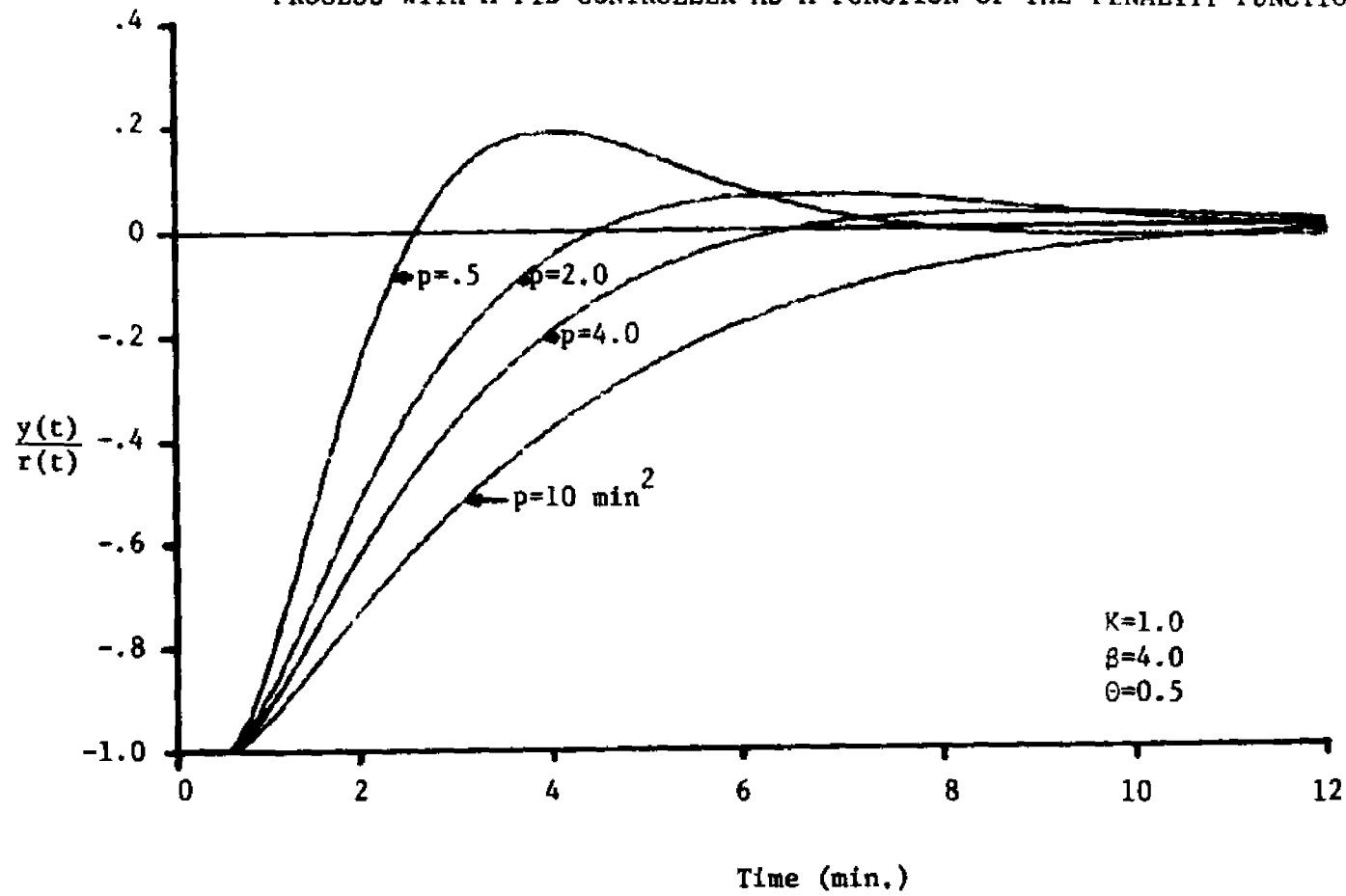
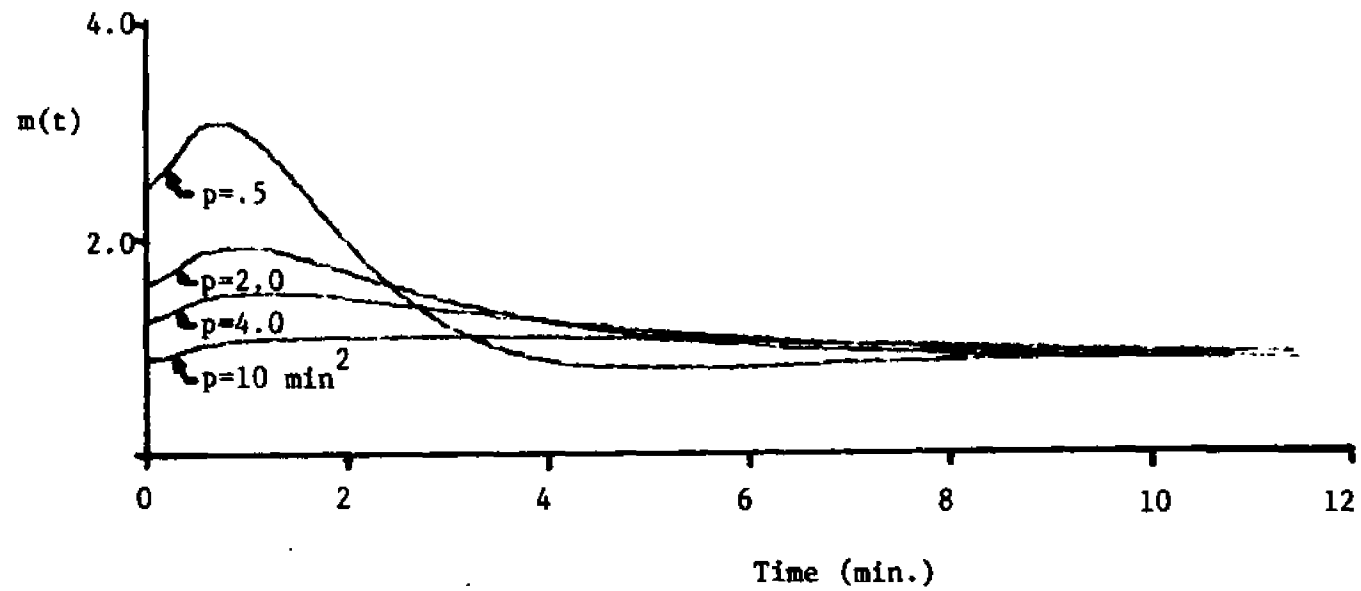


FIGURE 4-3

CONTROLLER OUTPUT AS A FUNCTION OF THE PENALTY FUNCTION



criteria established in Chapter III. And the process will again be considered to be represented by the second-order-lag-plus-dead-time transfer function. The process parameters that will be used to develop the tuning correlations will be the same as those used in Chapter III and Appendix F. Process models will also be as described in Chapter II.

Results For PI Controller

When the process reaction curve is fitted with a first-order-lag-plus-dead-time model and the dead-time is approximated by the first-order Taylor series expansion, a PI controller results. This was shown in Chapter II and Appendix E and the resulting tuning equations for the PI controller is given by:

$$G_c(s) = K_c \left[1 + \frac{1}{T_i} \cdot \frac{1}{s} \right]$$

$$\text{where: } KK_c = \frac{-\frac{t_0}{\tau} Je_{12} + Je_{22}}{r + \frac{t_0}{\tau} - \frac{t_0}{\tau} Je_{12} + Je_{22}}$$

$$\frac{\tau}{T_i} = \frac{-\frac{t_0}{\tau} Je_{11} + Je_{21} - \frac{t_0}{\tau} Je_{12} + Je_{22}}{-\frac{t_0}{\tau} Je_{12} + Je_{22}}$$

Je_{ij} in the controller parameter relationships are the elements of the steady-state solution matrix to the first-order nonlinear matrix Riccate equation. This was discussed in detail in Chapter II. Although a plot of p versus the model parameters is of little usefulness, it can serve to establish an order of magnitude of p which will result in a five percent overshoot performance. The correlation of r or $p/\tau^2 K^2$ (dimensionless) to the dimensionless parameter, τ/t_0 , of the first-order-lag-plus-dead-time model of the process is presented in

Figure 4-4. As can be seen from the figure, r is a strong function of the process parameters for small value of τ/t_0 but levels off as τ/t_0 increases.

The PI controller settings resulting from the application of the output regulator are presented in Figure 4-5. The plot is presented in terms of the dimensionless quantities KK_c , T_1/τ , and τ/t_0 . Therefore, knowing the parameters of the first-order-model, the controller gain, K_c , and reset time, T_1 , can be determined. Note that, as τ/t_0 increases, the controller gain increases and the dimensionless reset time decreases. This is what would be expected, since the larger the first-order time constant, the more highly overdamped the second-order process which requires a larger gain to make the system overshoot, but less integral action to keep it near its desired value.

Results for PID Controller

When the process reaction curve is fitted with a first-order-lag-plus-dead-time model of the process and the dead-time is approximated by the first-order Pade' approximation, which increases the order of the process model from first-order to second-order, a PID controller results. Also, if a second-order-lag-plus-dead-time model is fitted to the process reaction curve and the dead-time is approximated by the first-order Taylor series expansion, a PID controller results. Therefore two PID controllers result from the application of the output regulator to two different process models. This was shown in Chapter II and Appendix E. The resulting tuning equation for the PID controller, using the first-order-lag-plus-dead-time model is given by:

$$G_c(s) = K_c \left(1 + \frac{1}{T_1} \cdot \frac{1}{s} + T_d \cdot s \right)$$

FIGURE 4-4

CORRELATION OF THE TUNING PARAMETER FOR A

PI CONTROLLER RESULTING IN A 5% OVERSHOOT CRITERIA AS

A FUNCTION OF THE PARAMETERS OF A FIRST-ORDER-LAG-PLUS-DEAD-TIME MODEL

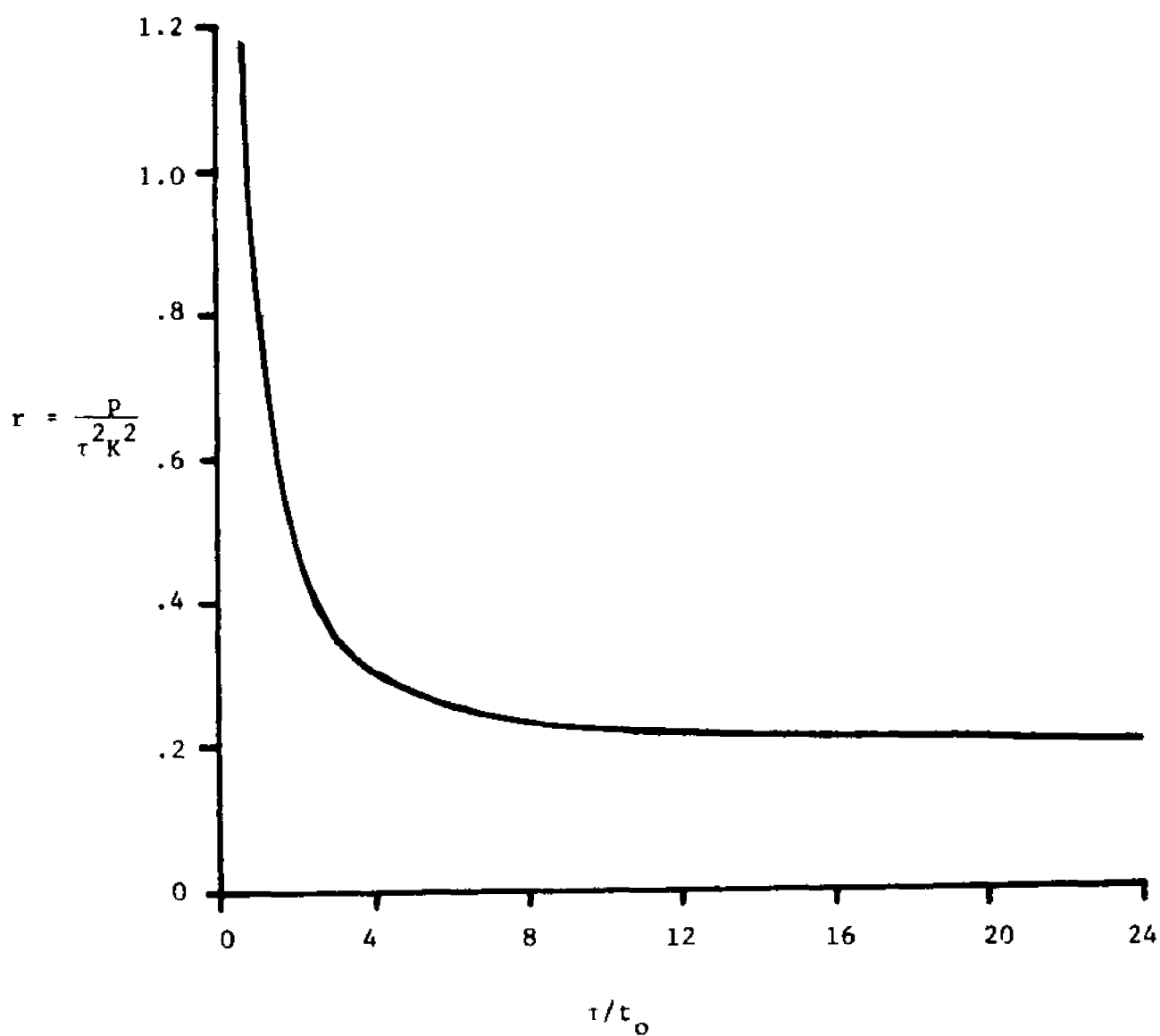
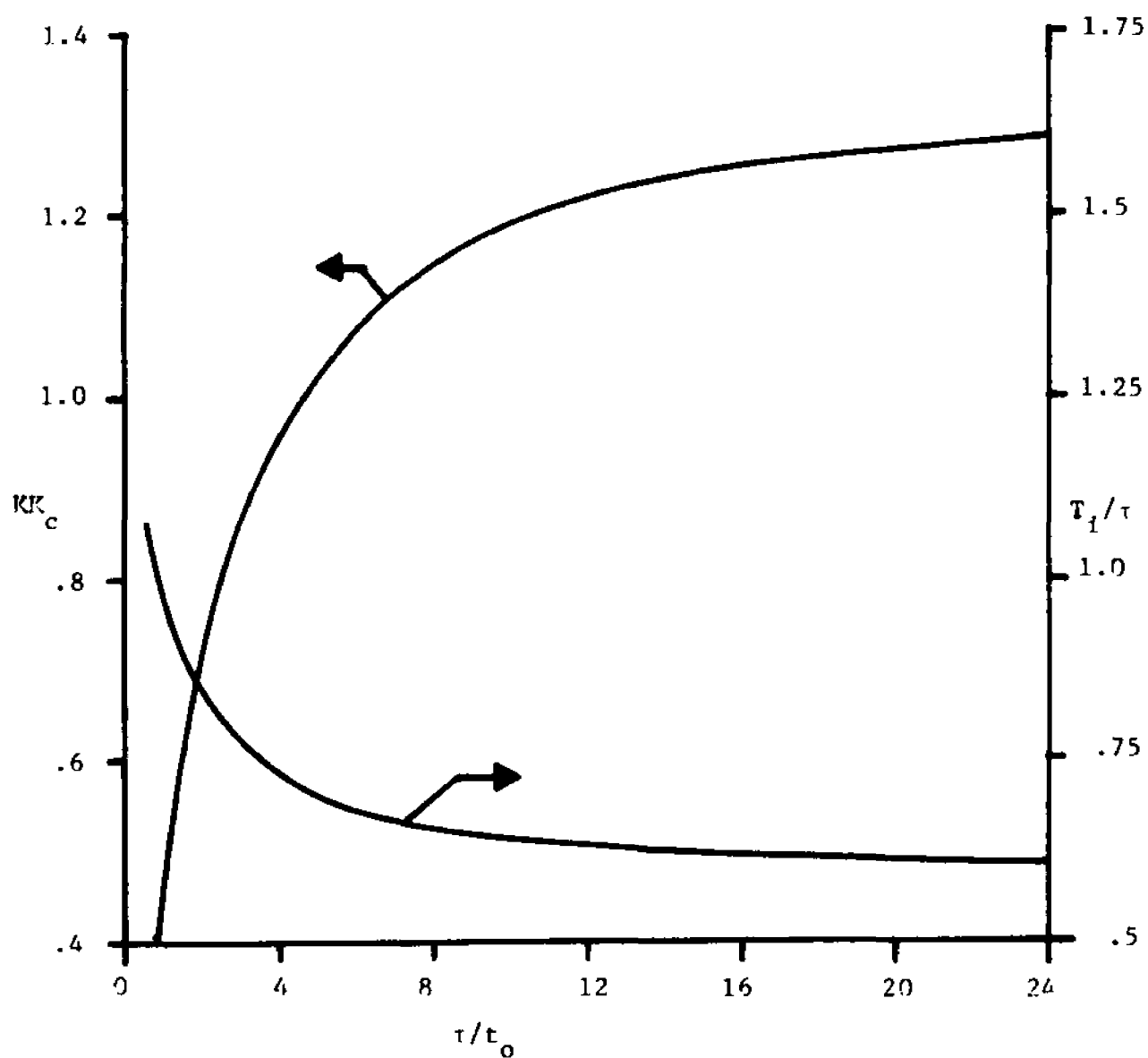


FIGURE 4-5
 OPTIMAL PI CONTROLLER TUNING PARAMETERS FOR 5%
 OVERSHOOT CRITERIA AS A FUNCTION OF THE FIRST-
 ORDER-LAG-PLUS-DEAD-TIME MODEL PARAMETERS



where

$$KK_c = \frac{(\frac{t_0}{\tau} + 4) J_{e32} + (\frac{t_0}{\tau} + 2) J_{e33}}{\frac{t_0}{\tau} J_{e32} + \frac{t_0}{\tau} J_{e33} + 2 r}$$

$$\frac{\tau}{T_1} = \frac{2(J_{e31} + J_{e32} + J_{e33})}{(\frac{t_0}{\tau} + 4) J_{e32} + (\frac{t_0}{\tau} + 2) J_{e33}}$$

$$\frac{T_d}{\tau} = \frac{\frac{t_0}{\tau} (J_{e32} + J_{e33})}{(\frac{t_0}{\tau} + 4) J_{e32} + (\frac{t_0}{\tau} + 2) J_{e33}}$$

The resulting tuning equation for the PID controller using the second-order-lag-plus-dead-time model is given by

$$G_c(s) = K_c \left[1 + \frac{1}{T_1} \cdot \frac{1}{s} + T_d \cdot s \right]$$

where

$$KK_c = \frac{(1+\beta\theta)J_{e32} + \beta J_{e33}}{r + \theta^2 J_{e32} + \theta J_{e33}}$$

$$\frac{1}{\sqrt{c} T_1} = \frac{J_{e31} + \theta J_{e32} + J_{e33}}{(1+\beta\theta)J_{e32} + \beta J_{e33}}$$

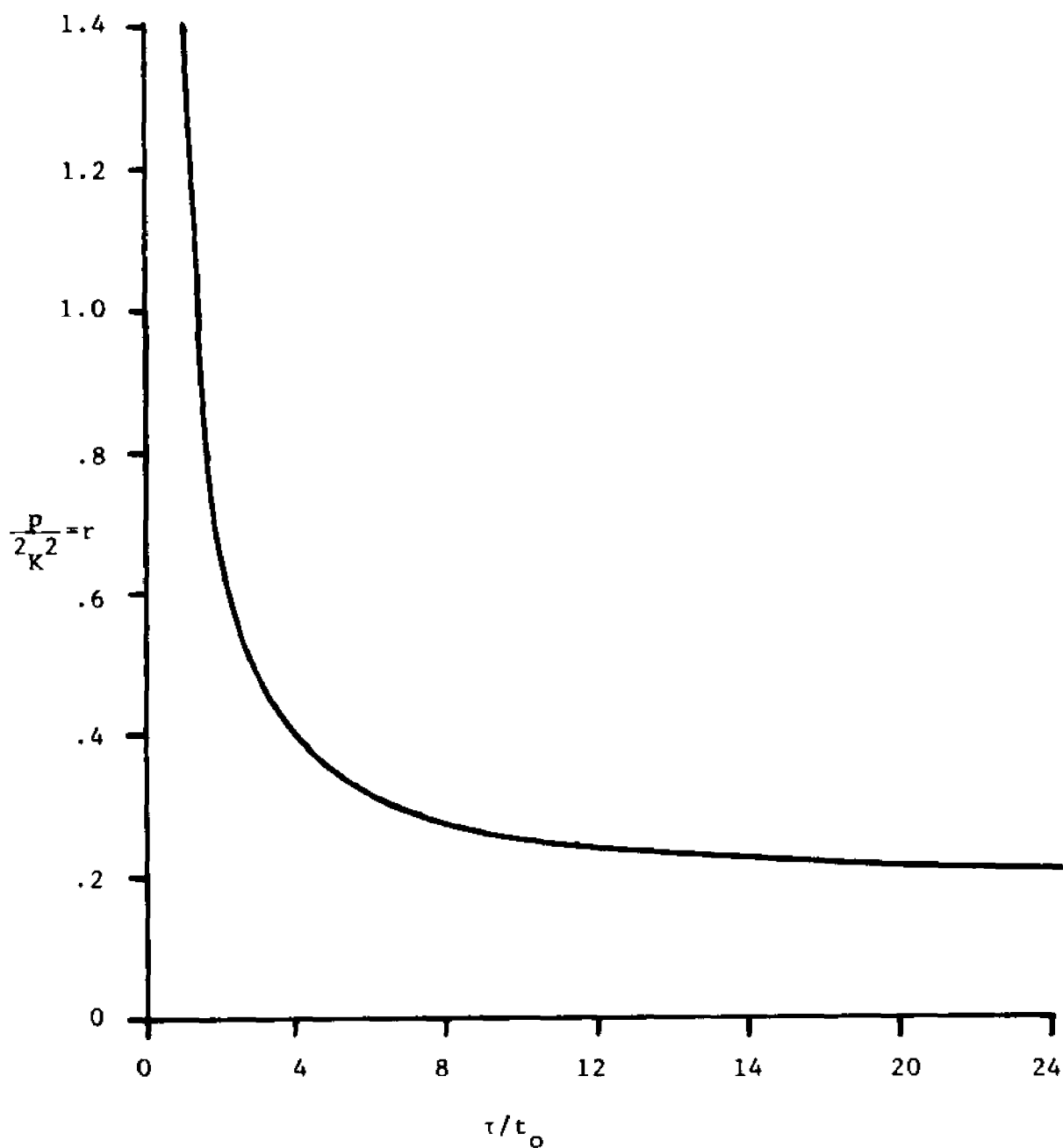
$$\sqrt{c} T_d = \frac{\theta J_{e32} + J_{e33}}{(1+\beta\theta) J_{e32} + \beta J_{e33}}$$

Since the tuning correlations will be developed as a function of the model parameters, two sets of plots will be required.

The results for the PID controller resulting from the first-order model will be considered first. r or $p/\tau^2 K^2$ (dimensionless) is presented as a function of τ/t_0 , the dimensionless parameter of the first-order-lag-plus-dead-time model in Figure 4-6. This plot is similar to Figure 4-4 with the only difference being r is slightly

FIGURE 4-6

CORRELATION OF THE TUNING PARAMETER FOR A PID CONTROLLER
 RESULTING IN A 5% OVERSHOOT CRITERIA AS A FUNCTION OF THE
 PARAMETERS OF A FIRST-ORDER-LAG-PLUS-DEAD-TIME MODEL



larger for the PID controller. The resulting controller settings are presented in Figures 4-7 and 4-8. Again the plots are presented in the dimensionless quantities KK_c , T_i/τ , and T_d/τ . The controller gain is presented as a function of the dimensionless model parameter τ/t_0 in Figure 4-7 and the dimensionless reset time and dimensionless preact time are presented as a function of τ/t_0 in Figure 4-8. Here the results are as expected, as τ/t_0 increases, the controller gain increases and the dimensionless reset and preact times decrease.

The results for the PID controller resulting from the second-order model are presented in the next four figures. r or $\frac{PC}{K^2}$ (dimensionless) is presented as a function of the dimensionless quantities β and θ of the second-order-lag-plus-dead-time model in Figure 4-9. The quantity β is a damping parameter and is equal to twice the damping ratio, ζ , and θ is the dimensionless dead-time. From this figure it can be seen that for any value of β , as the dead-time increases, p increases. Also note that as the damping parameter β , increases, p increases. The resulting dimensionless controller gain, KK_c , the dimensionless reset time, $T_i\sqrt{c}$, and the dimensionless preact time, $T_d\sqrt{c}$, are presented in Figures 4-10, 4-11, and 4-12 respectively as a function of β and θ . It is important to note that the dimensionless preact time $T_d\sqrt{c}$ is primarily a function of the damping parameter, β , and for all practical purposes is not a function of θ , the dimensionless dead-time. This is not the case for the dimensionless gain (Figure 4-10) and the dimensionless reset time (Figure 4-11), as they are both strong functions of the damping parameter and dead-time.

Using the results presented here, a PID controller can be tuned by knowing the parameters of either a first-order-lag-plus-

FIGURE 4-7
OPTIMAL PID CONTROLLER GAIN FOR 5% OVERSHOOT CRITERIA AS
A FUNCTION OF THE PARAMETERS OF A FIRST-ORDER
LAG-PLUS-DEAD-TIME MODEL

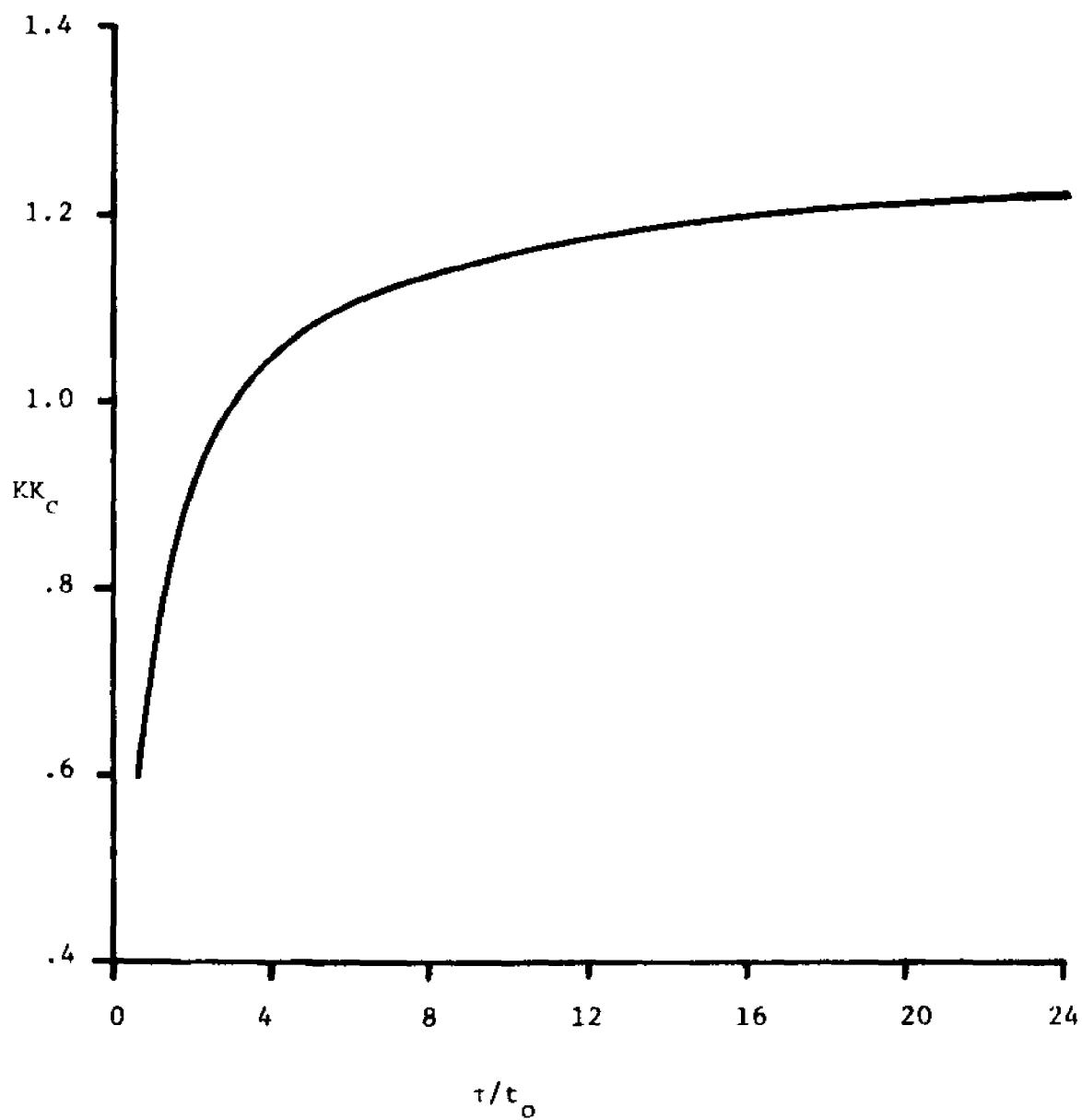


FIGURE 4-8
 OPTIMAL PID RESET AND PREACT TIMES FOR 5% OVERSHOOT
 CRITERIA AS A FUNCTION OF THE PARAMETERS OF A
 FIRST-ORDER-LAG-PLUS-DEAD-TIME MODEL

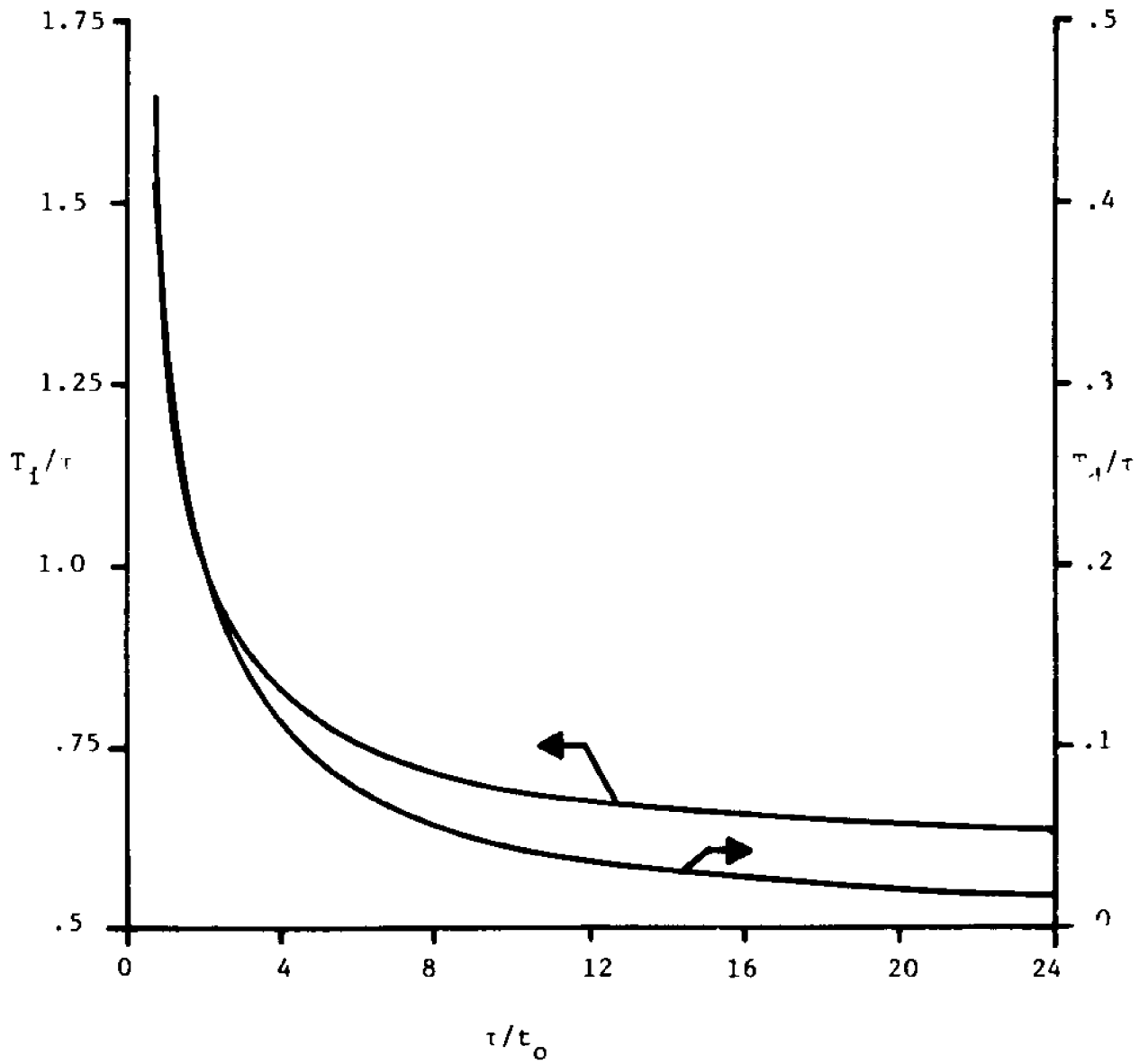


FIGURE 4-9

CORRELATION OF THE TUNING PARAMETER FOR A PID CONTROLLER
 RESULTING IN A 5% OVERSHOOT CRITERIA AS A FUNCTION OF
 THE PARAMETERS OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME MODEL

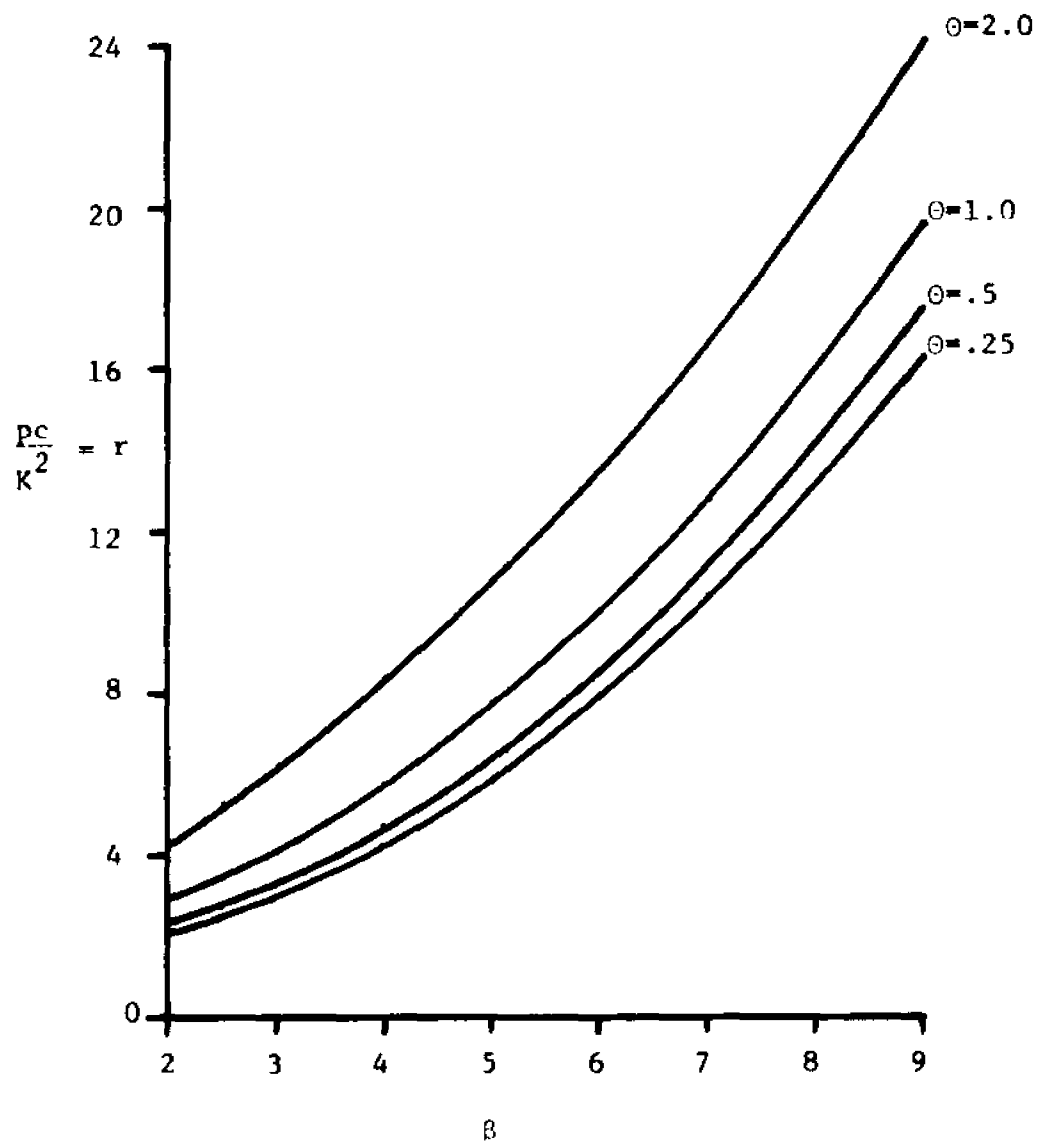


FIGURE 4-10
OPTIMAL PID CONTROLLER GAIN FOR 5% OVERSHOOT CRITERIA
AS A FUNCTION OF THE PARAMETERS OF A SECOND-ORDER-
LAG-PLUS-DEAD-TIME MODEL

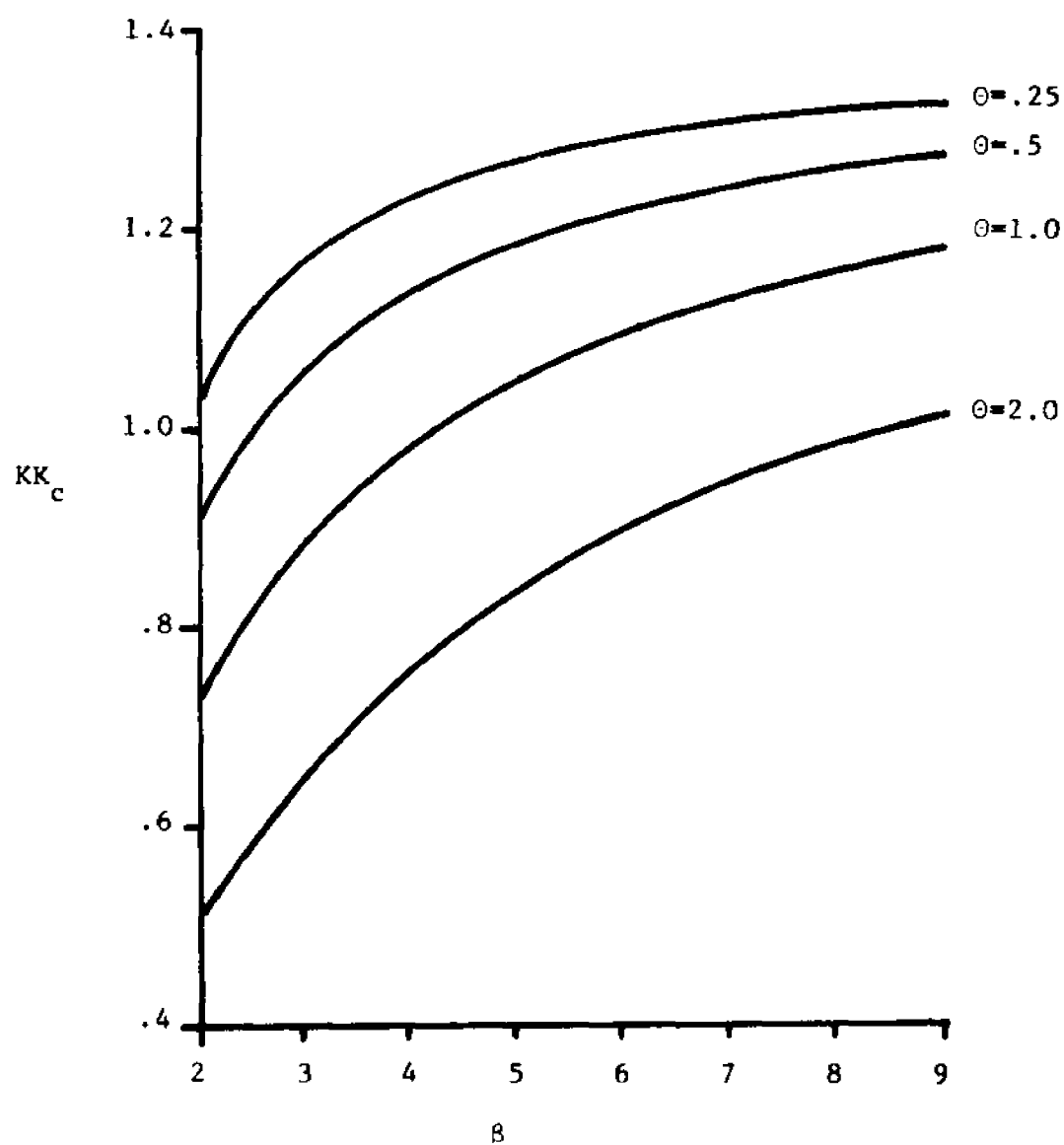


FIGURE 4-11
OPTIMAL PID RESET TIME FOR 5% OVERSHOOT CRITERIA AS A
FUNCTION OF THE PARAMETERS OF A SECOND-ORDER

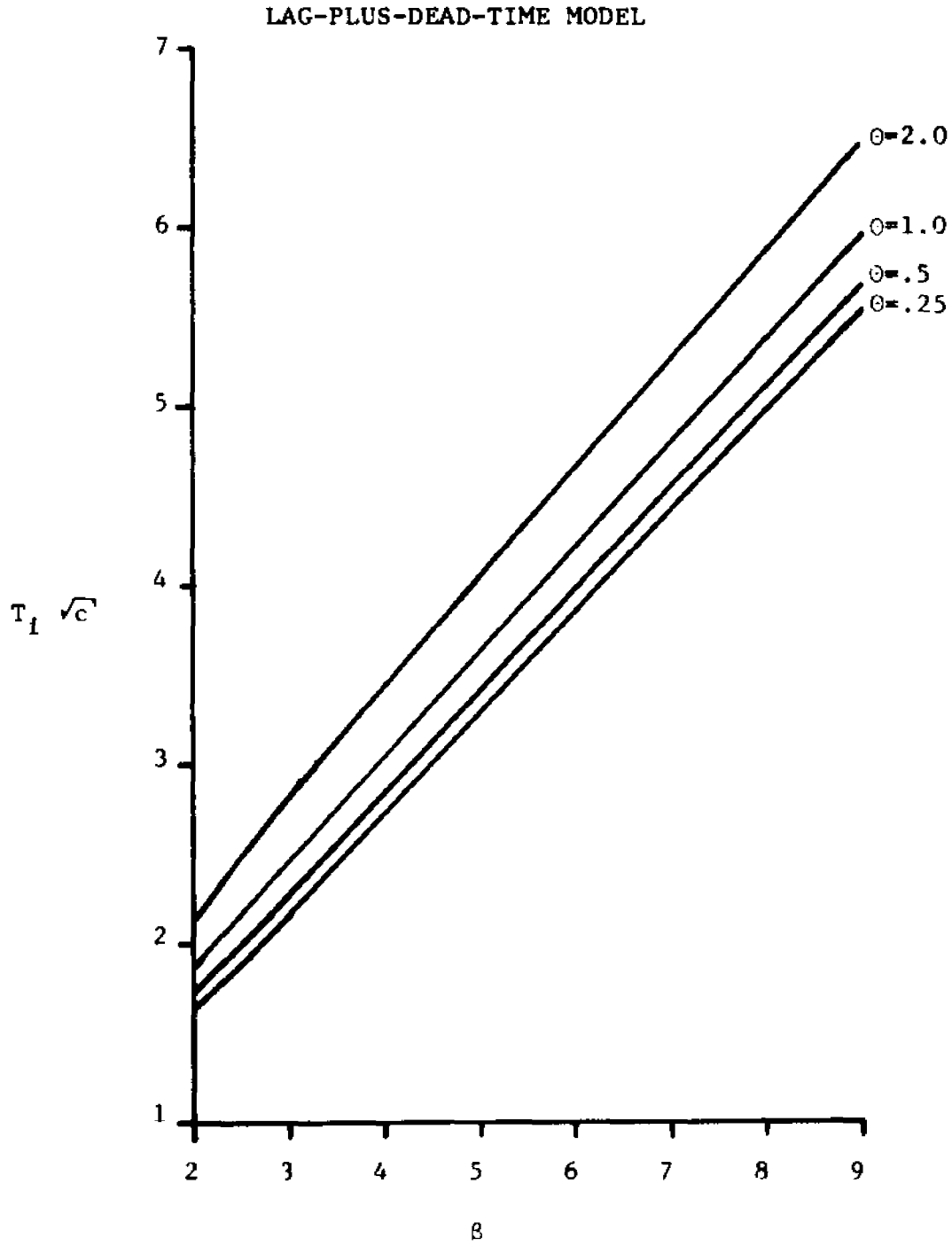


FIGURE 4-12
 OPTIMAL PID PREACT TIME FOR 5% OVERSHOOT CRITERIA AS A
 FUNCTION OF THE PARAMETERS OF A SECOND-ORDER-
 LAG-PLUS-DEAD-TIME MODEL

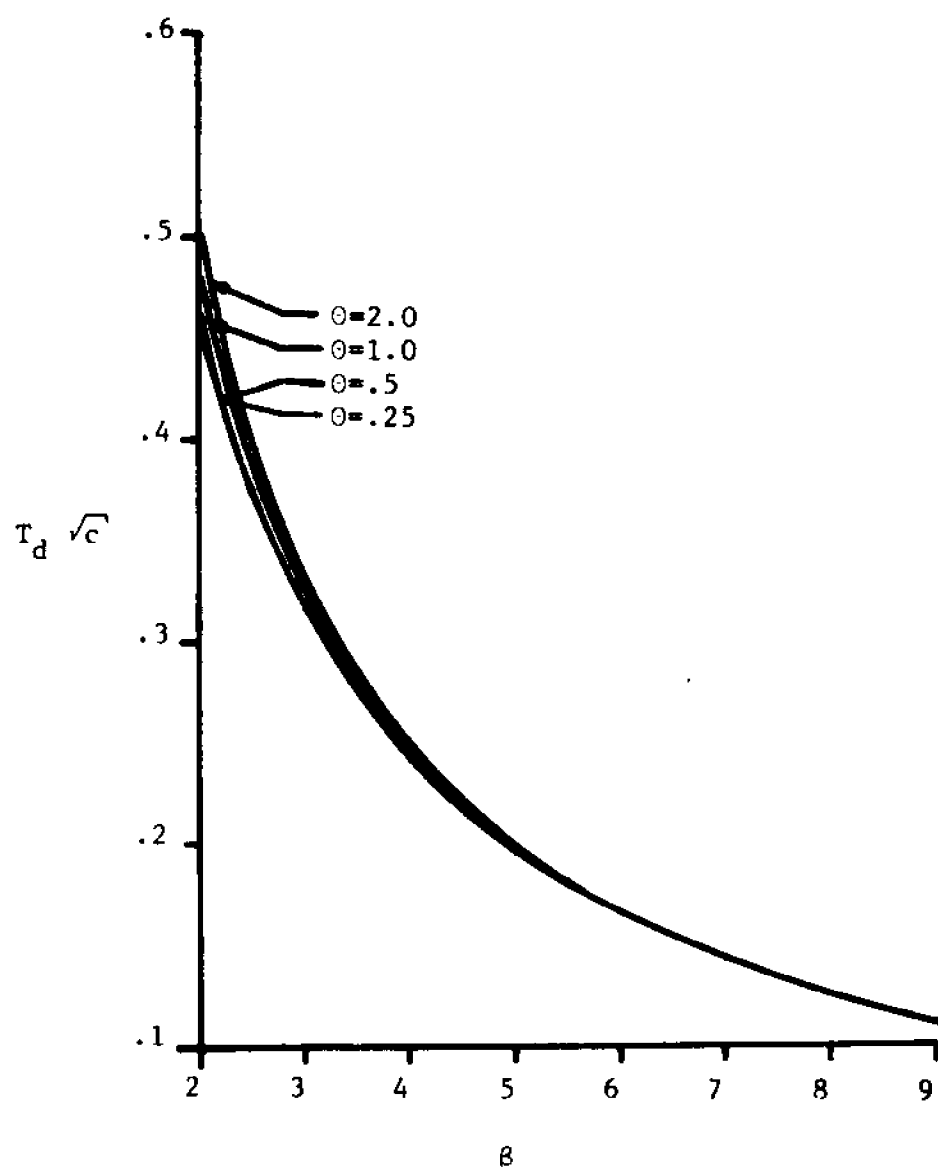
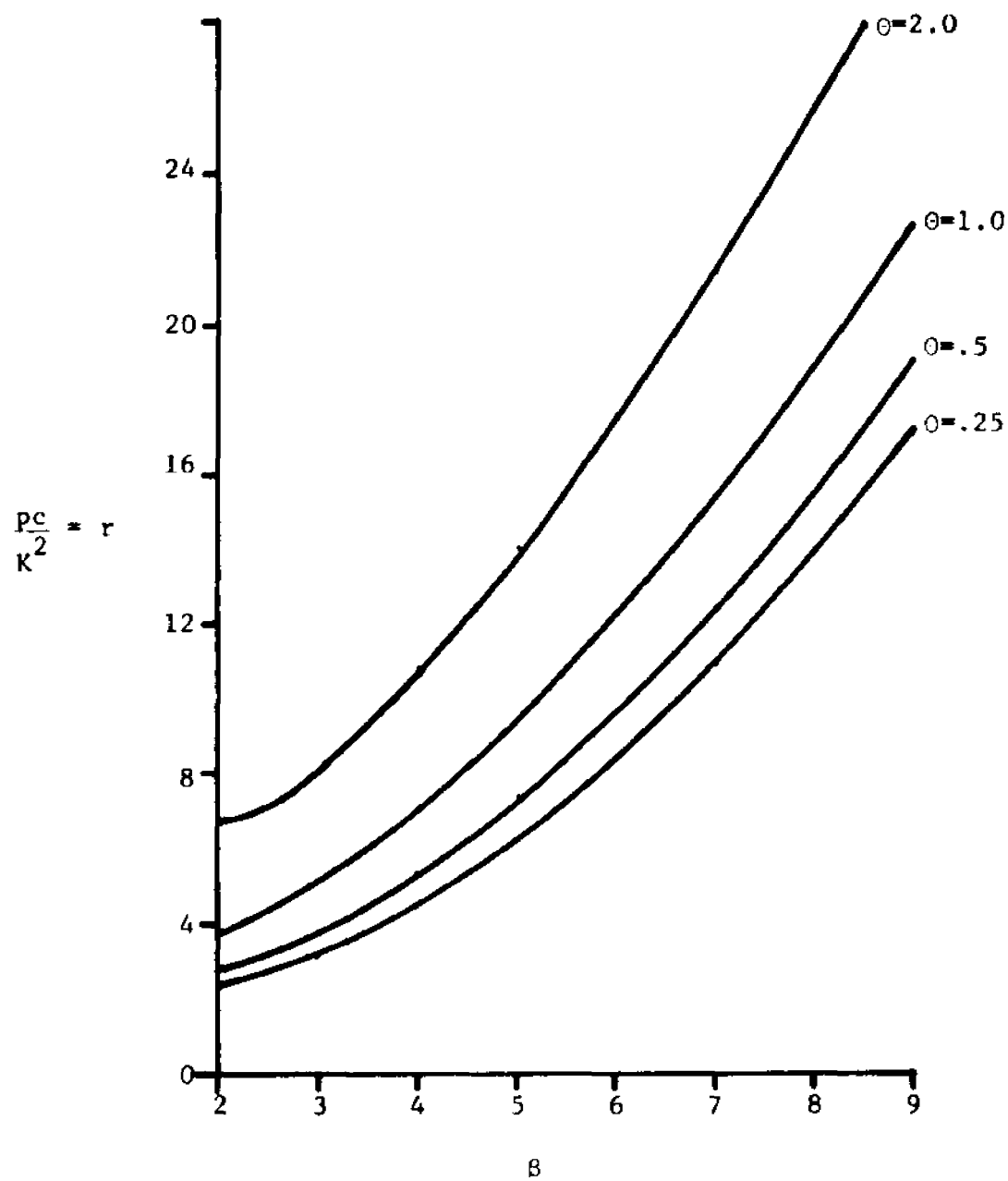


FIGURE 4-13

CORRELATION OF THE TUNING PARAMETER FOR A PIDD²
 CONTROLLER RESULTING IN A 5% OVERSHOOT CRITERIA AS A FUNCTION
 OF THE PARAMETERS OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME MODEL



dead-time model or second-order-lag-plus-dead-time model of the process.

Results for PIDD² Controller

When the process reaction curve is fitted with a second-order-lag-plus-dead-time model of the process and the dead-time is approximated with the first-order Pade' approximation, which increases the order of the process model from second-order to third-order, a PIDD² or proportional-plus-integral-plus-derivative-plus derivative squared controller results. This was shown in Chapter II and Appendix E. The resulting tuning equations for the PIDD² controller are given by:

$$G_c(s) = K_c \left(1 + \frac{1}{T_1} \cdot \frac{1}{s} + T_{d1} \cdot s + T_{d2} \cdot s^2 \right)$$

where

$$KK_c = \frac{J_{e42} + \left(\beta + \frac{\theta}{2}\right)(J_{e43} + J_{e44})}{r + \frac{\theta}{2}(J_{e43} + J_{e44})}$$

$$\frac{1}{\sqrt{c} T_1} = \frac{J_{e41} + J_{e43} + J_{e44}}{J_{e42} + \left(\beta + \frac{\theta}{2}\right)(J_{e43} + J_{e44})}$$

$$\sqrt{c} T_{d1} = \frac{\left(2 + \frac{\theta\beta}{2}\right) J_{e43} + \left(1 + \frac{\theta\beta}{2}\right) J_{e44}}{J_{e42} + \left(\beta + \frac{\theta}{2}\right)(J_{e43} + J_{e44})}$$

$$c T_{d2} = \frac{\frac{\theta}{2}(J_{e43} + J_{e44})}{J_{e42} + \left(\beta + \frac{\theta}{2}\right)(J_{e43} + J_{e44})}$$

Although this may be considered a theoretical controller, the contribution of the additional derivative term will be examined in order to determine the effect of this term on the system response.

r or pc/K^2 (dimensionless) is presented as a function of the dimensionless parameters β and θ in Figure 4-13. This plot is similar to Figure 4-9 for the PID controller resulting from the

second-order model but the values of r are slightly larger for the $PIDD^2$ controller. The resulting controller parameters are presented as follows:

Figure 4-14: $KK_c = f(\beta, \theta)$

Figure 4-15: $\sqrt{c} T_1 = f(\beta, \theta)$

Figure 4-16; $\sqrt{c} T_{d1} = f(\beta, \theta)$

Figure 4-17: $c T_{d2} = f(\beta, \theta)$

Immediately it should be noted that the coefficient of the derivative-squared term is small for the overdamped cases examined. Unlike the second-order model PID controller, all controller parameters for the $PIDD^2$ controller are strong functions of the dimensionless dead-time. Therefore knowing the parameters of a second-order-lag-plus-dead-time model of the process, the parameters of the $PIDD^2$ controller, which will result in a five percent overshoot criteria, can be determined.

All of the preceding plots were obtained by the following procedure:

- 1) Search over p to obtain the controller settings which would produce the desired 5% overshoot for a given set of process parameters. To accomplish this it is necessary to:
 - a) Obtain the solution of the matrix-Riccati equation by relaxation technique.
 - b) Obtain the solution of the closed-loop system equations by Runge-Kutta routine.
 - c) Iterate on percent overshoot by Secant method to obtain the desired response.
- 2) Repeat 1) for many different sets of process parameters to

FIGURE 4-14
OPTIMAL PIDD² CONTROLLER GAIN FOR 5% OVERSHOOT CRITERIA
AS A FUNCTION OF THE PARAMETERS OF A SECOND-
ORDER-LAG-PLUS-DEAD-TIME MODEL

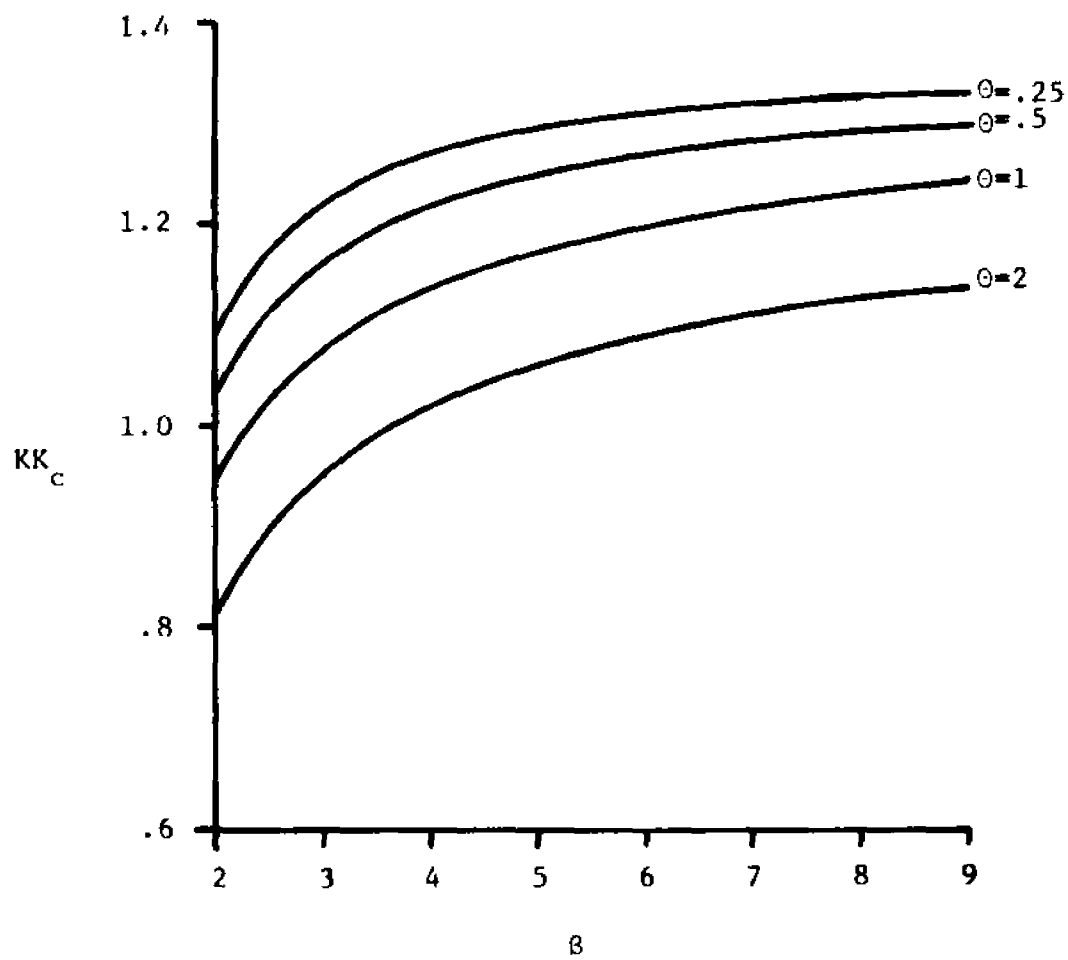


FIGURE 4-15
 OPTIMAL PID² CONTROLLER RESET TIME FOR 5% OVERSHOOT
 CRITERIA AS A FUNCTION OF THE PARAMETERS OF A

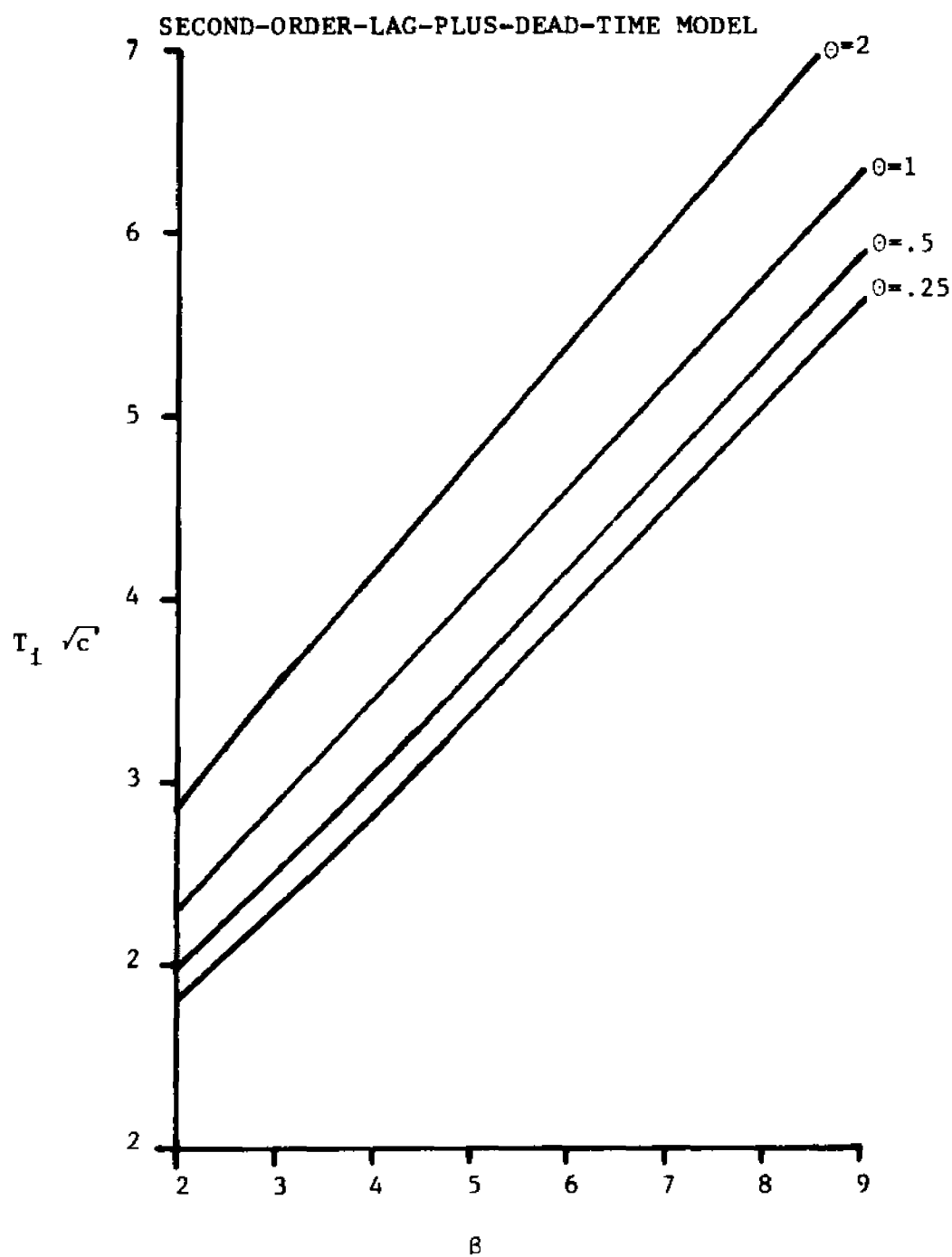


FIGURE 4-16
 OPTIMAL PIDD² CONTROLLER PREACT TIME FOR 5%
 OVERSHOOT CRITERIA AS A FUNCTION OF THE PARAMETERS
 OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME MODEL

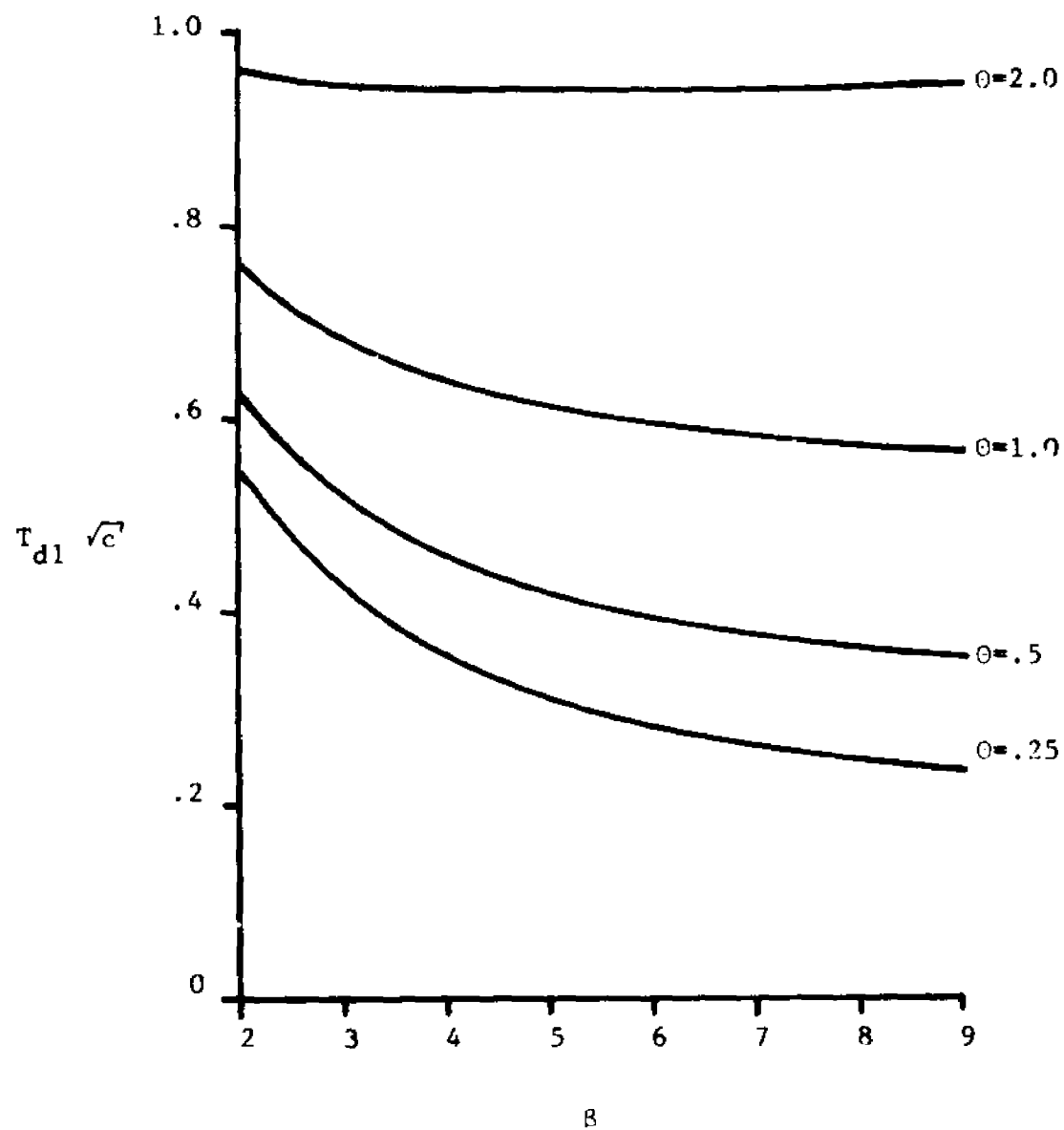
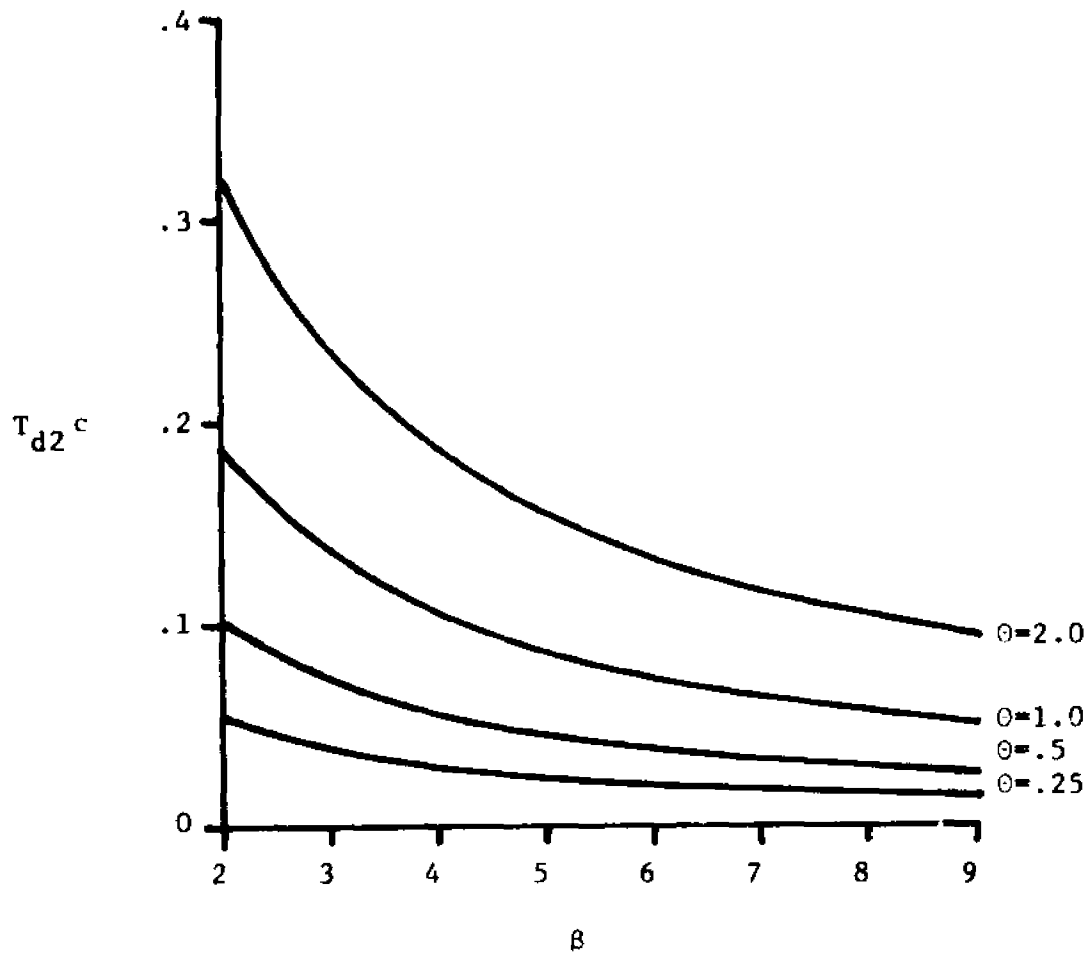


FIGURE 4-17
 OPTIMAL PIDD² CONTROLLER PREACT TIME - SQUARED
 FOR 5% OVERSHOOT CRITERIA AS A FUNCTION OF THE
 PARAMETERS OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME MODEL



obtain the necessary data points to establish a correlation.

3) Plot the results versus the parameters of the process model.

As pointed out earlier, the controller settings are determined as a function of the model parameters, the steady state solution of the matrix Riccati equation, and the penalty function.

Now in general, if the parameters of a first-order model are known, a PI controller may be tuned by using Figures 4-5 or a PID controller may be tuned by using Figure 4-7 and 4-8. If the parameters of a second-order model are known a PID controller may be tuned by using Figures 4-10, 4-11 and 4-12.

Comparison With Other Tuning Techniques

Comparison of the results of the output regulator tuning will be with the techniques considered in Chapter III, that of Ziegler-Nichols (1), quarter decay ratio, Rovira (2), minimum IAE, and controller synthesis, 5 percent overshoot criteria. A comparison of the tuning relationships is not feasible because simplified tuning equations were not obtained for the output regulator. However, plots of the tuning parameters as a function of the parameters of the process model can be compared.

The PI controller gains and dimensionless reset times are presented in Figures 4-18 and 4-19, respectively, as a function of the dimensionless first-order model parameter, τ/t_0 . These figures show that the output regulator results in the smallest controller gain and a dimensionless integral action which is similar to Rovira's and controller synthesis. Note, the reset time is independent of the model dead-time for a value of τ/t_0 greater than 8.0.

Some typical responses which could be expected by using these

FIGURE 4-18
 PI CONTROLLER GAINS AS A FUNCTION OF THE PARAMETERS
 OF A FIRST-ORDER-LAG-PLUS-DEAD-TIME MODEL

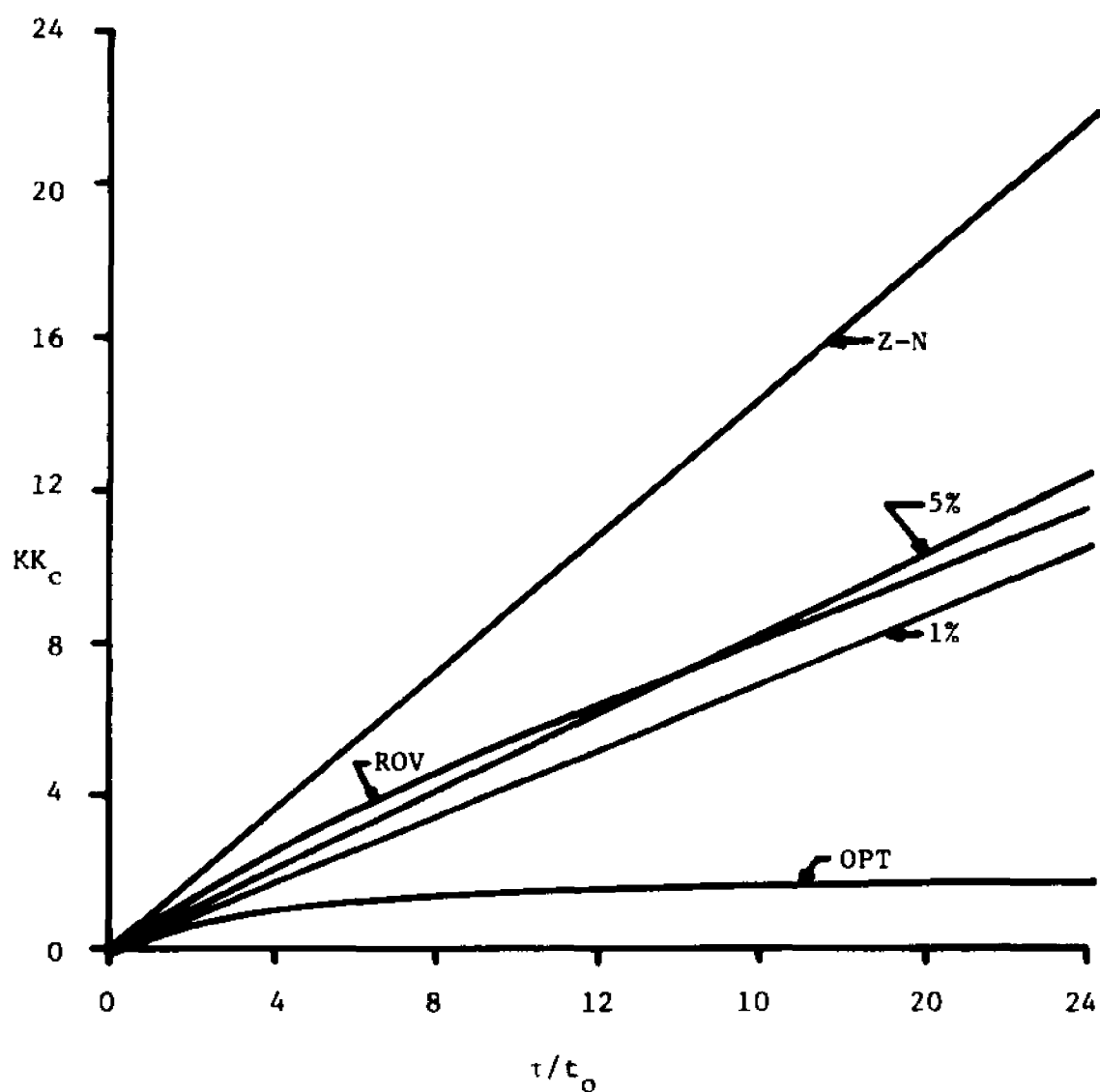
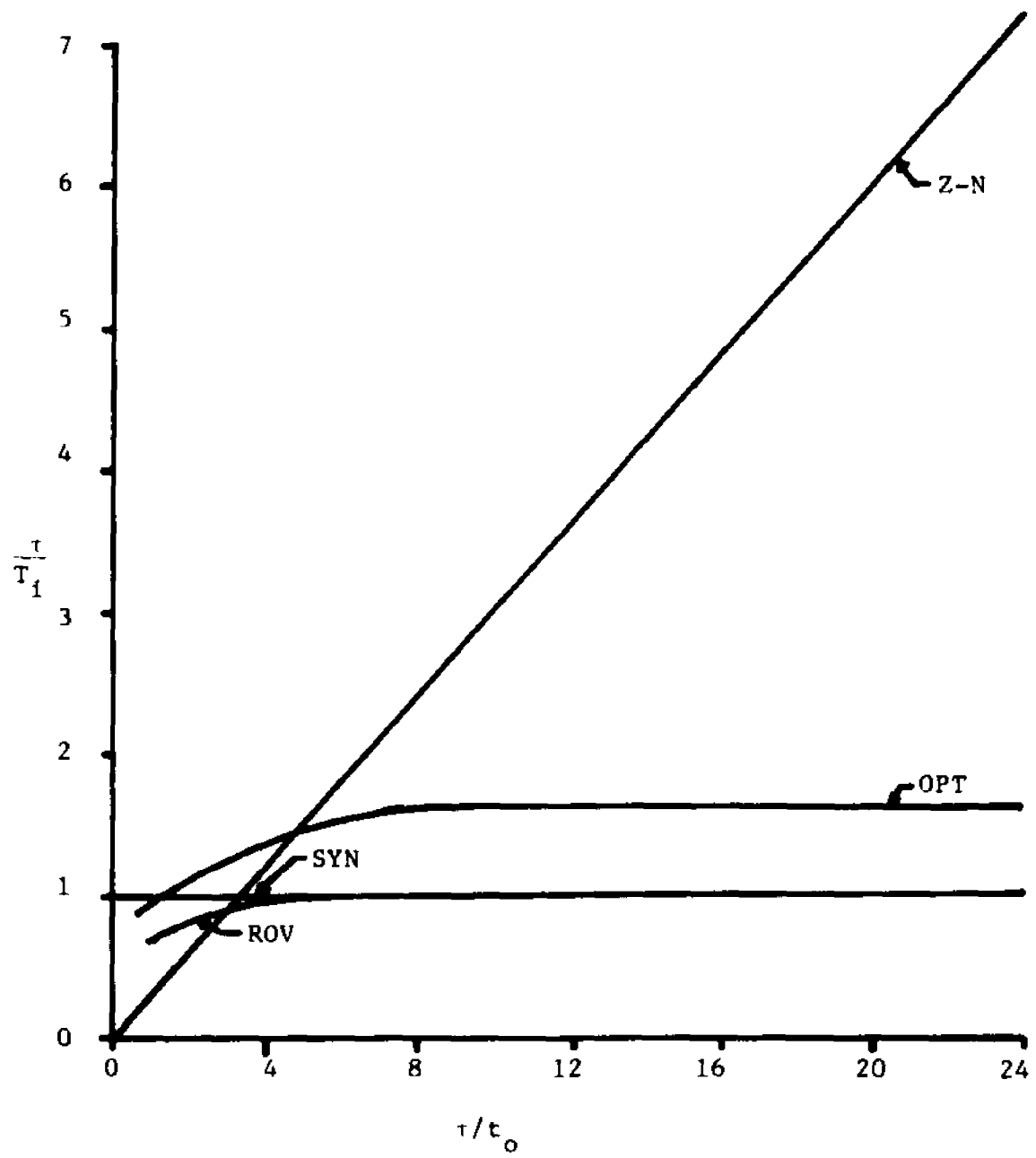


FIGURE 4-19

PI CONTROLLER RESET TIME AS A FUNCTION OF THE PARAMETERS
OF A FIRST-ORDER-LAG-PLUS-DEAD-TIME MODEL



PI controller tuning relationships are presented in the next four figures. The process parameters and parameters of the first-order-lag-plus-dead-time model of the process are presented in Table 4-2 for each of the figures. The PI controller settings for the respective tuning technique for each of the figures are presented in Table 4-3. The closed-loop responses for the critically damped process with short and long dead-time are presented in Figures 4-20 and 4-21, respectively. The closed-loop responses for the overdamped process with short and long dead-time are presented in Figures 4-22 and 4-23, respectively. The different techniques will be denoted as follows:

Z-N - Ziegler-Nichols (1) - quarter decay ratio criteria

ROV - Rovira (2) - minimum IAE criteria

SYN - Controller synthesis - 5% overshoot criteria

OPT - Output regulator - 5% overshoot criteria

These figures show that, while the optimal regulator tuning exhibit good dynamic behavior (does not oscillate), the responses are significantly slower for the overdamped processes. The sluggishness is caused by the penalty function for the valve movement. The controller outputs are presented in Figure 4-24 for the responses shown in Figure 4-23. This figure shows that the valve does not move rapidly and does not move far from its steady-state value.

The PID controller gains, dimensionless reset time, and dimensionless preact time are presented in Figures 4-25, 4-26 and 4-27, respectively. The optimal regulator gain, Figure 4-25, is considerably less than the other techniques gains and is almost constant for all values of τ/τ_0 . The dimensionless reset time is less than the synthesis

TABLE 4-2
PROCESS AND MODEL PARAMETERS

Figure	Process Parameters				First-order Model Parameters			
	K	$b(\text{min}^{-1})$	$c(\text{min}^{-2})$	$t_o(\text{min})$	K	$\tau(\text{min})$	$t_o(\text{min})$	τ/t_o
4-20	1.0	2.0	1.0	0.25	1.0	1.638	0.758	2.16
4-21	1.0	2.0	1.0	1.0	1.0	1.638	1.508	1.09
4-22	1.0	4.0	1.0	0.25	1.0	3.726	0.531	7.01
4-23	1.0	4.0	1.0	1.0	1.0	3.726	1.281	2.91

TABLE 4-3
PI CONTROLLER SETTINGS

Figure	Technique	KK_c	T_1 (min)
4-20	Z-N	1.945	2.525
	ROV	1.472	1.883
	SYN (5%)	1.137	1.639
	OPTR	.743	1.364
4-21	Z-N	.978	5.025
	ROV	.814	2.268
	SYN (5%)	.571	1.639
	OPTR	.531	1.605
4-22	Z-N	6.311	1.770
	ROV	4.055	3.831
	SYN (5%)	3.688	3.731
	OPTR	1.131	2.513
4-23	Z-N	2.617	4.274
	ROV	1.900	4.098
	SYN (5%)	1.530	3.731
	OPTR	0.891	2.882

FIGURE 4-20

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY DAMPED
PROCESS WITH SHORT DEAD-TIME FOR A PI CONTROLLER

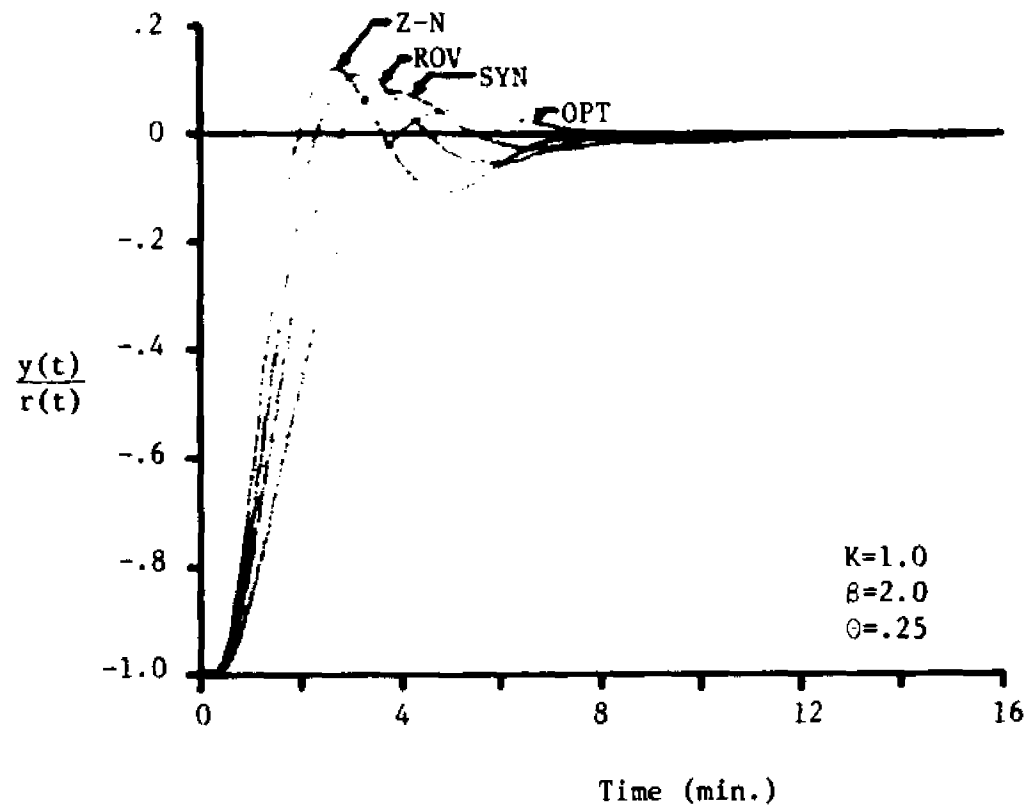


FIGURE 4-21

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY DAMPED PROCESS
WITH LONG DEAD-TIME FOR A PI CONTROLLER

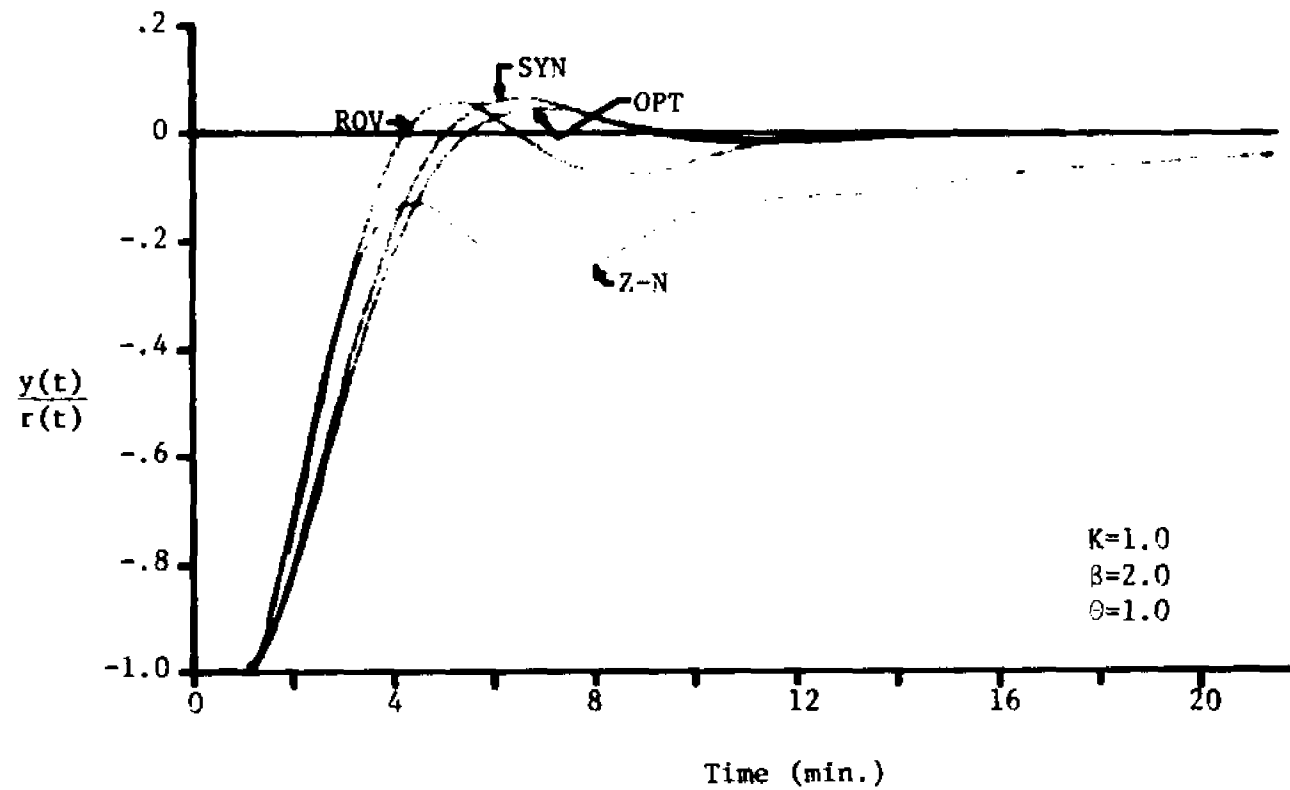


FIGURE 4-22

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED
PROCESS WITH SHORT DEAD-TIME FOR A PI CONTROLLER

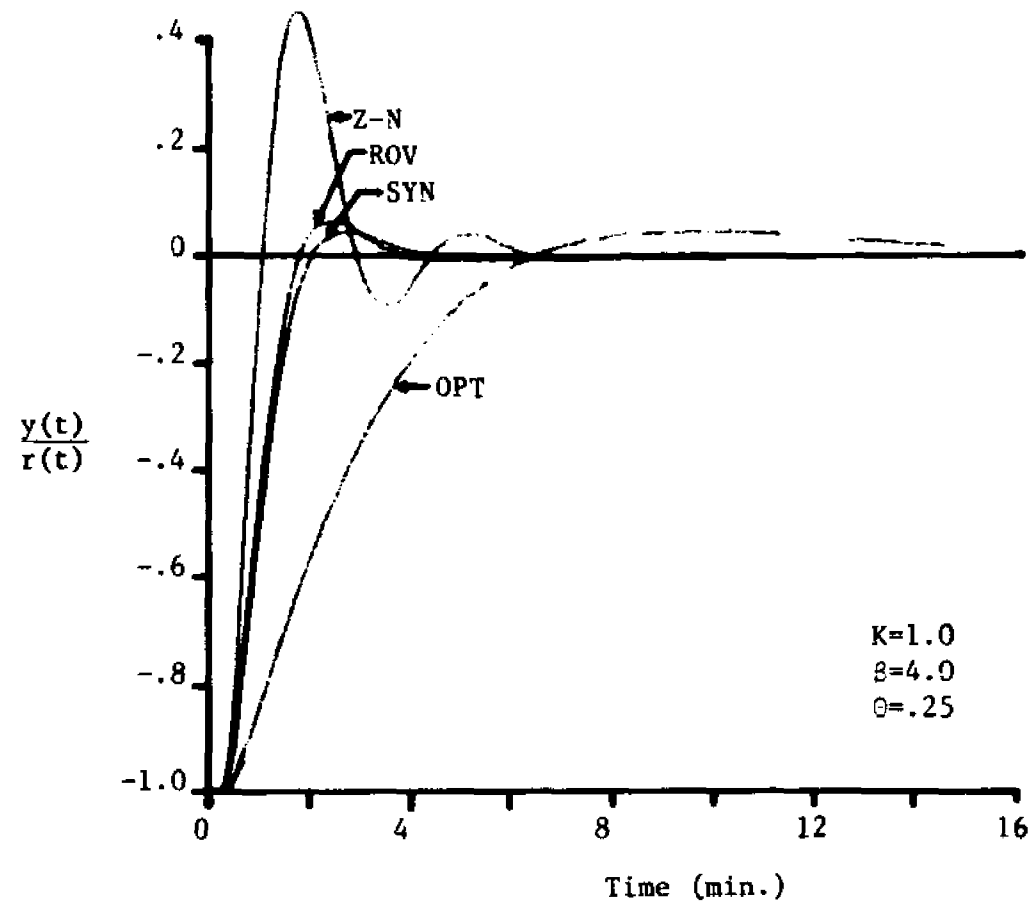


FIGURE 4-23

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED
PROCESS WITH LONG DEAD-TIME FOR A PI CONTROLLER

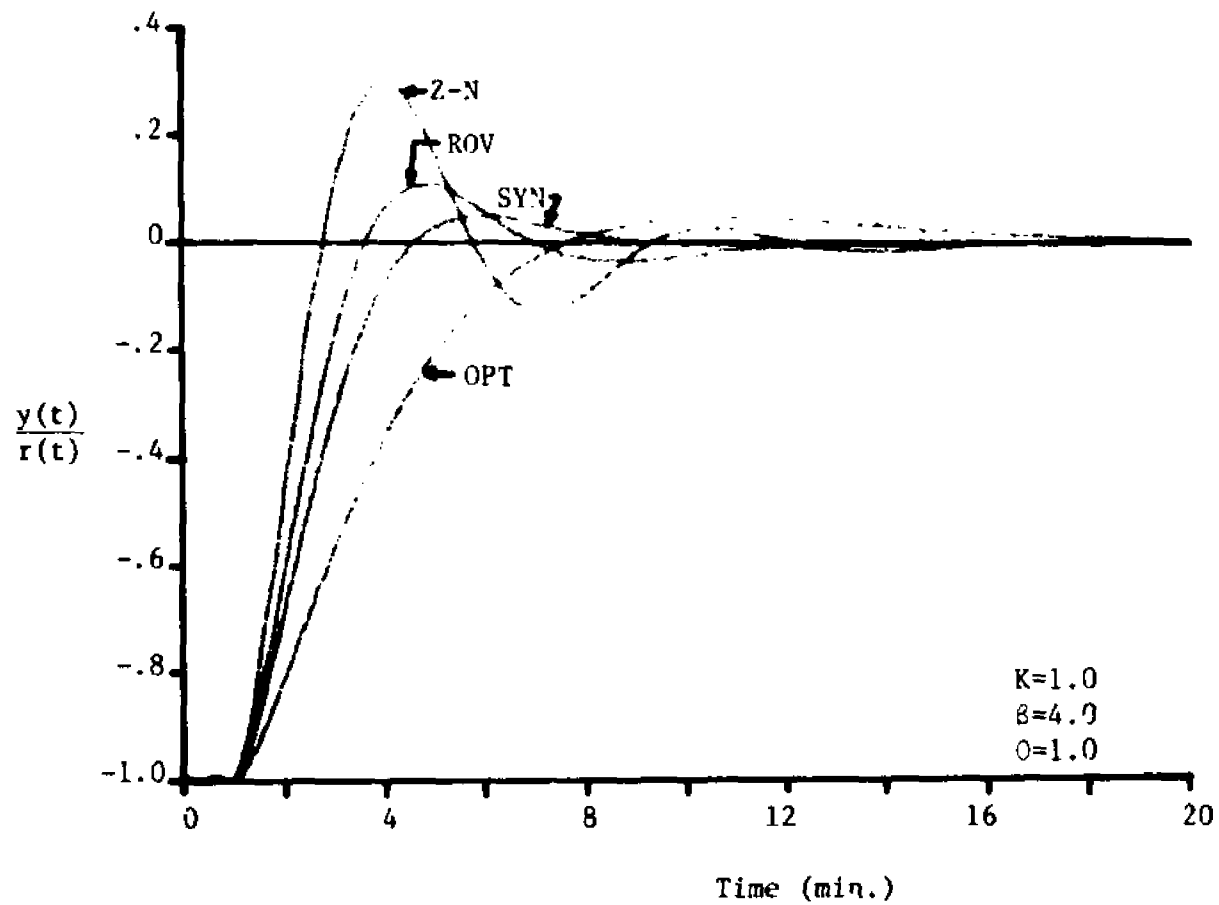


FIGURE 4-24
CONTROLLER OUTPUT OF A SECOND-ORDER OVERDAMPED
PROCESS WITH LONG DEAD-TIME FOR A PI CONTROLLER

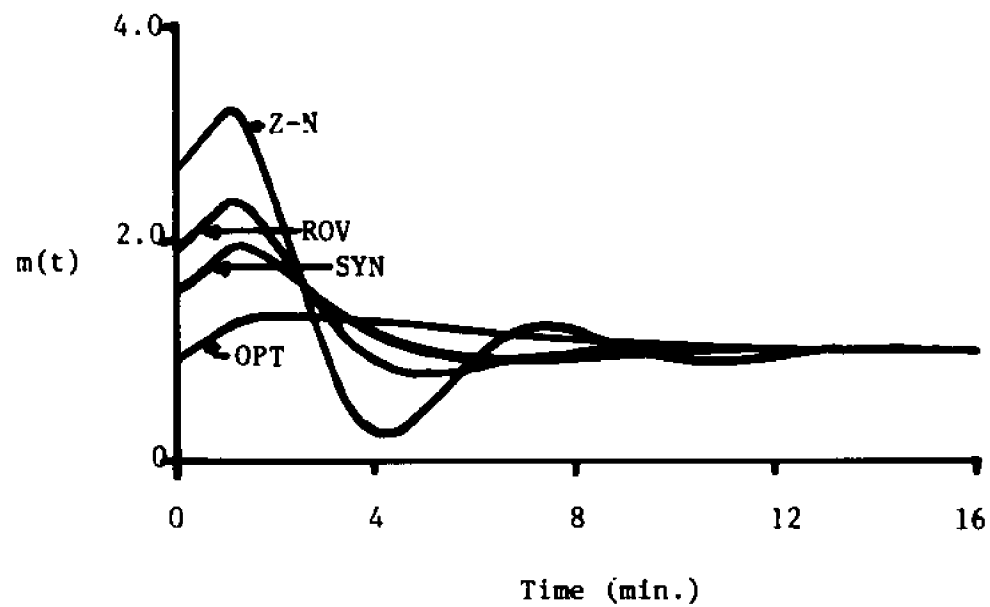


FIGURE 4-25
 PID CONTROLLER GAINS AS A FUNCTION OF THE PARAMETERS OF A
 SECOND-ORDER-LAG-PLUS-DEAD-TIME MODEL

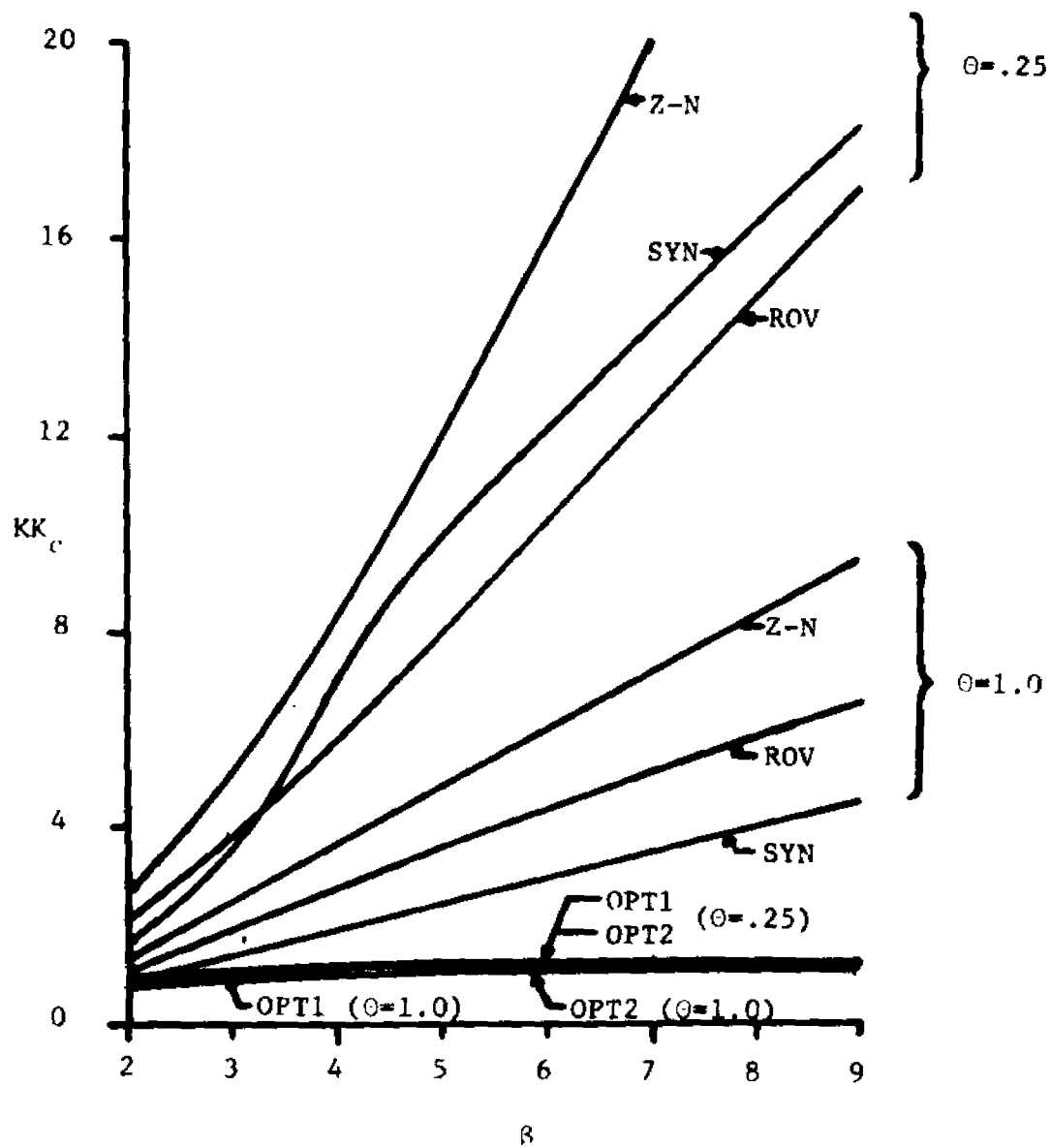


FIGURE 4-26

PID CONTROLLER RESET TIME AS A FUNCTION OF THE PARAMETERS
OF A SECOND-ORDER-LAG-PLUS-DEAD-TIME MODEL

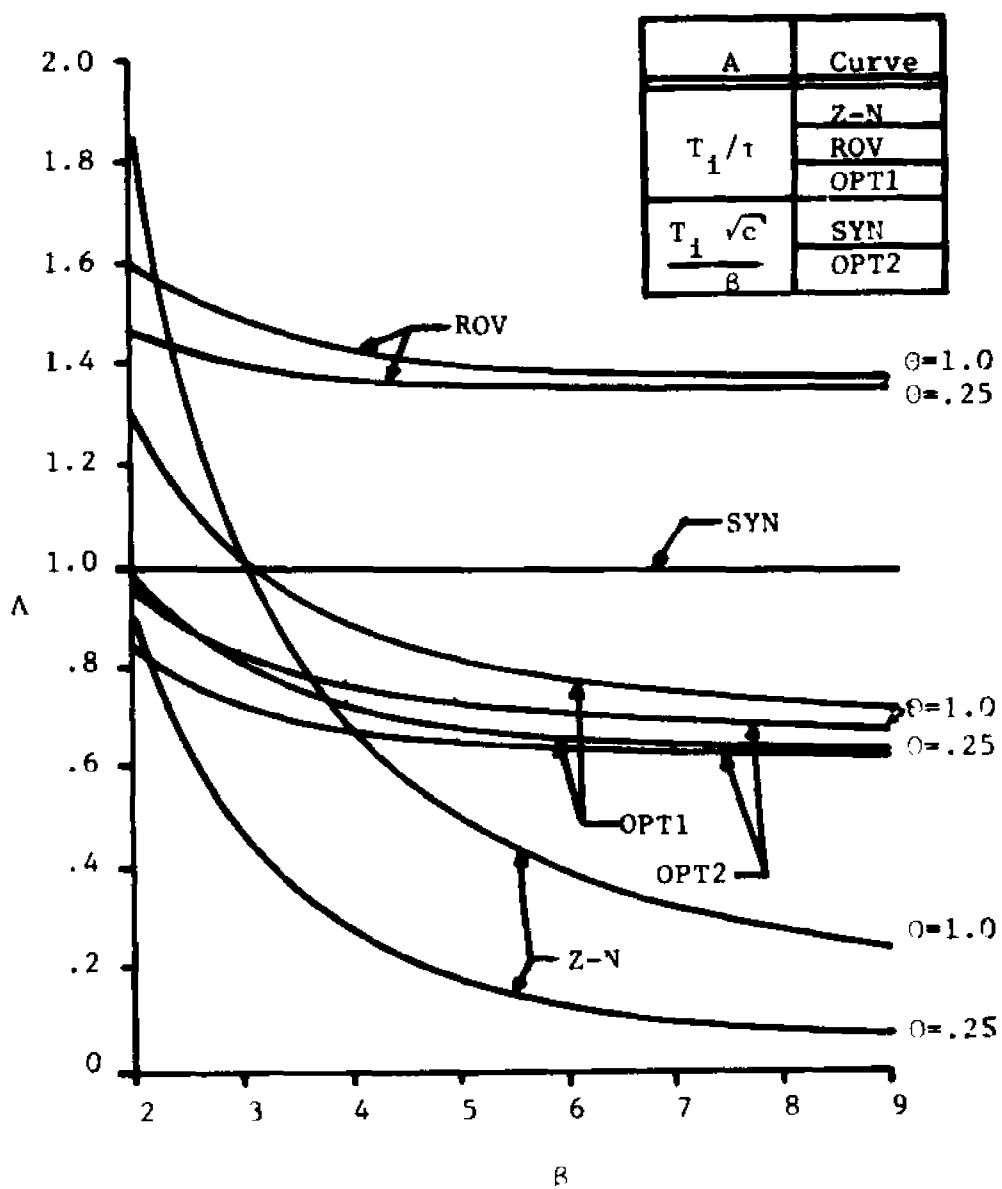
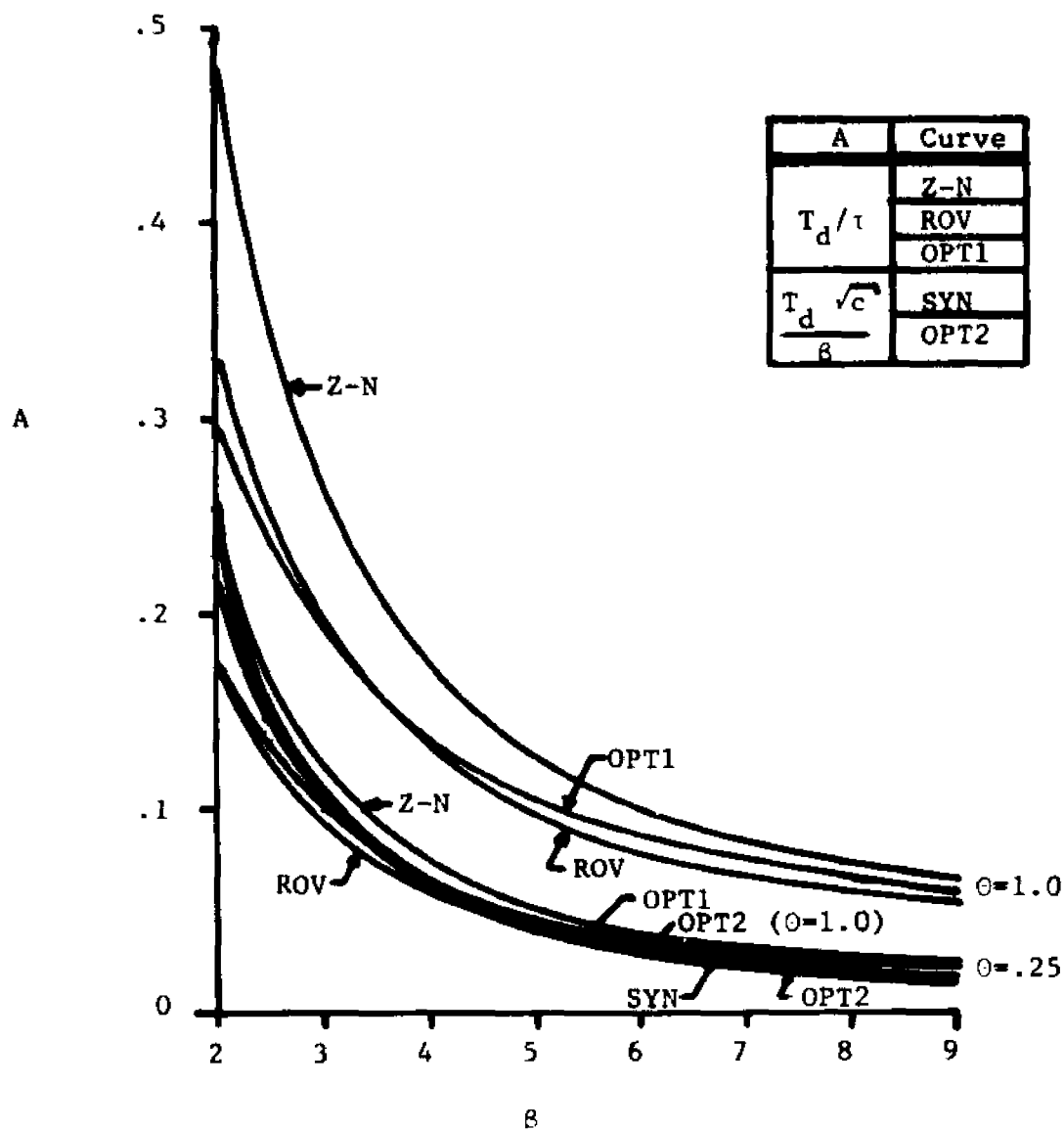


FIGURE 4-27

PID CONTROLLER PREACT TIMES AS A FUNCTION OF THE PARAMETERS
OF A SECOND-ORDER-PLUS-DEAD-TIME MODEL



dimensionless reset time but greater than the quarter decay ratio reset time. The preact time, Figure 4-27, is approximately the same for all three techniques.

Some typical closed-loop responses which could be expected by using these tuning techniques are presented in the next four figures. The process parameters and the parameters of the first-order-lag-plus-dead-time model of the process will be the same as those presented in Table 4-2. The PID controller settings for the respective tuning techniques for each of the figures are presented in Table 4-4. The closed-loop responses for the critically damped process with short and long dead-time are presented in Figures 4-28 and 4-29, respectively. The closed-loop responses for the overdamped process with short and long dead-time are presented in Figures 4-30 and 4-31, respectively. Each response curve is labeled as described earlier with the following exceptions:

OPT1 - Output regulator (based on first-order model) - 5% overshoot criteria.

OPT2 - Output regulator (based on second-order model) - 5% overshoot criteria.

These figures show that while the optimal regulator tuning again exhibits good dynamic behavior (does not oscillate) the response is significantly slower for the overdamped cases. Again this sluggishness is the result of slow valve movement as shown by Figure 4-32 which presents the controller output for the responses obtained in Figure 4-31.

Discussion

The Pade' approximation is generally considered to be a good

TABLE 4-5

PID CONTROLLER SETTINGS

Figure	Technique	K_c	T_i (min.)	T_d (min.)
4-28	Z-N	2.594	1.515	.378
	ROV	2.122	2.410	.282
	SYN (5%)	1.510	2.0	.500
	OPT1	1.001	1.616	.290
	OPT2	1.032	1.675	.452
4-29	Z-N	1.304	3.012	.753
	ROV	1.167	2.638	.529
	SYN (5%)	.8	2.0	.5
	OPT1	.862	2.110	.484
	OPT2	.732	1.905	.475
4-30	Z-N	8.415	1.062	0.265
	ROV	5.901	5.155	.220
	SYN (5%)	3.080	4.0	.250
	OPT1	1.231	2.717	.240
	OPT2	1.233	2.717	.242
4-31	Z-N	3.490	2.564	.641
	ROV	2.746	5.348	.488
	SYN (5%)	1.844	4.0	.25
	OPT1	1.102	3.322	.517
	OPT2	.983	3.067	.244

FIGURE 4-28

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY DAMPED

PROCESS WITH SHORT DEAD-TIME FOR A PID CONTROLLER

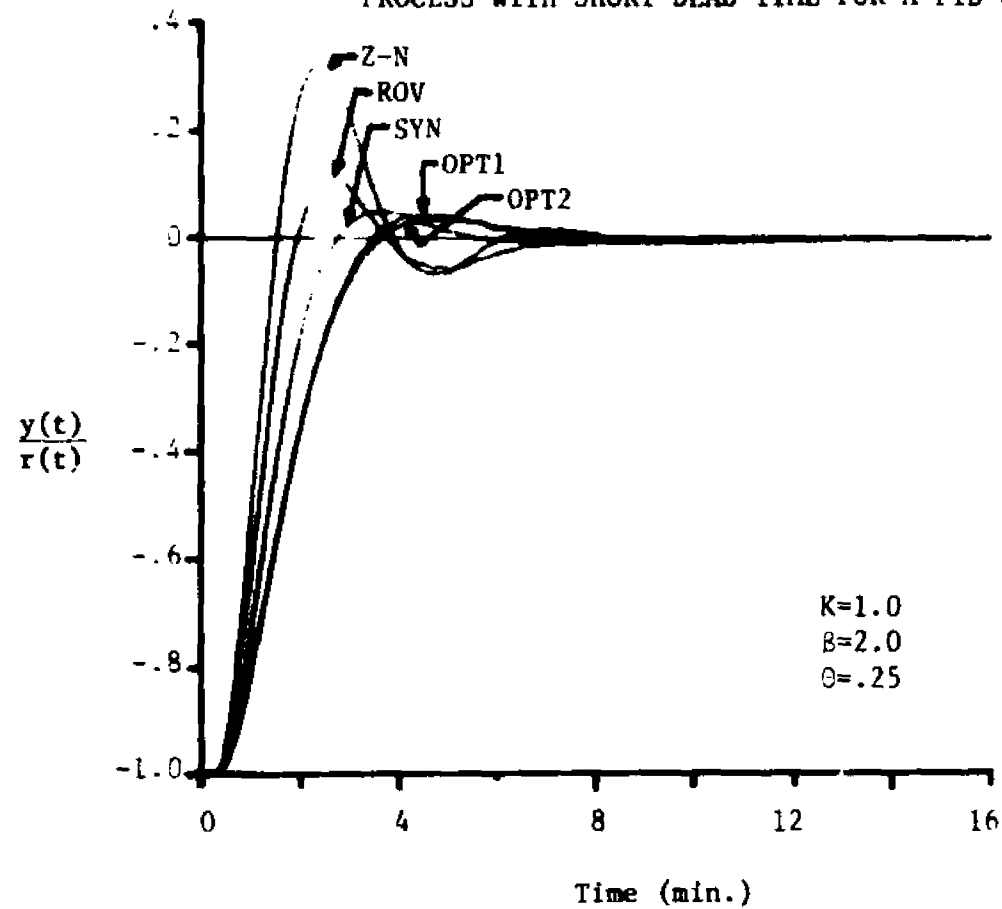


FIGURE 4-29

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER CRITICALLY DAMPED
PROCESS WITH LONG DEAD-TIME FOR A PID CONTROLLER

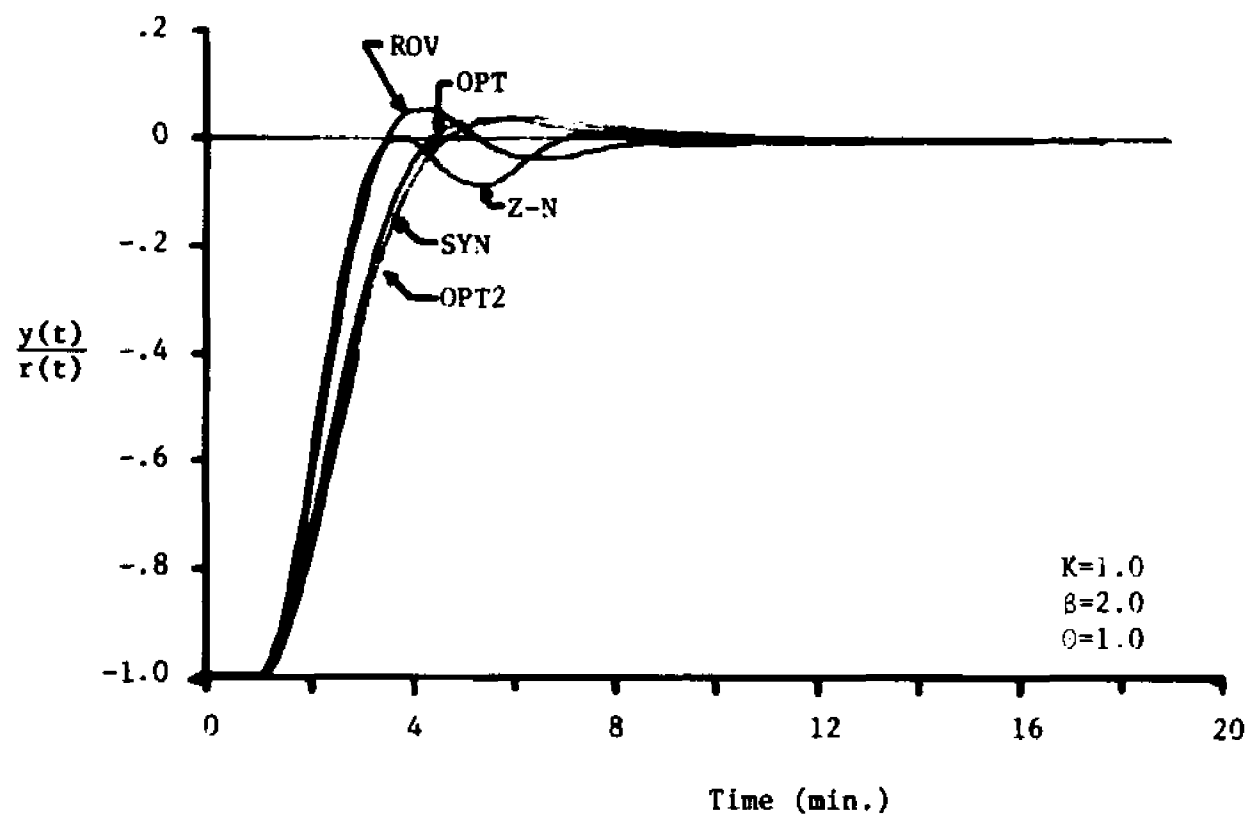


FIGURE 4-30

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED

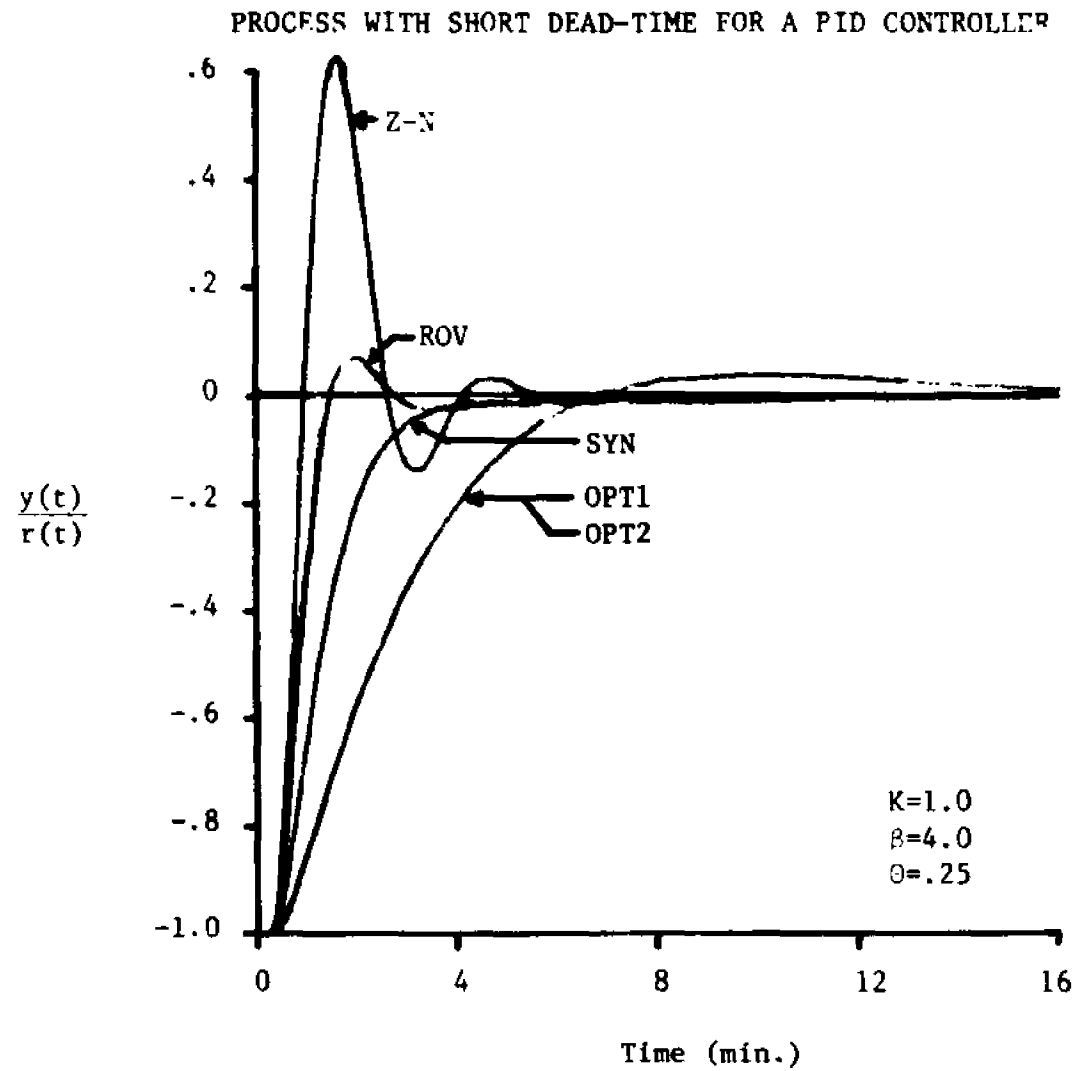


FIGURE 4-31

TYPICAL CLOSED-LOOP RESPONSES OF A SECOND-ORDER OVERDAMPED PROCESS
WITH LONG DEAD-TIME FOR A PID CONTROLLER

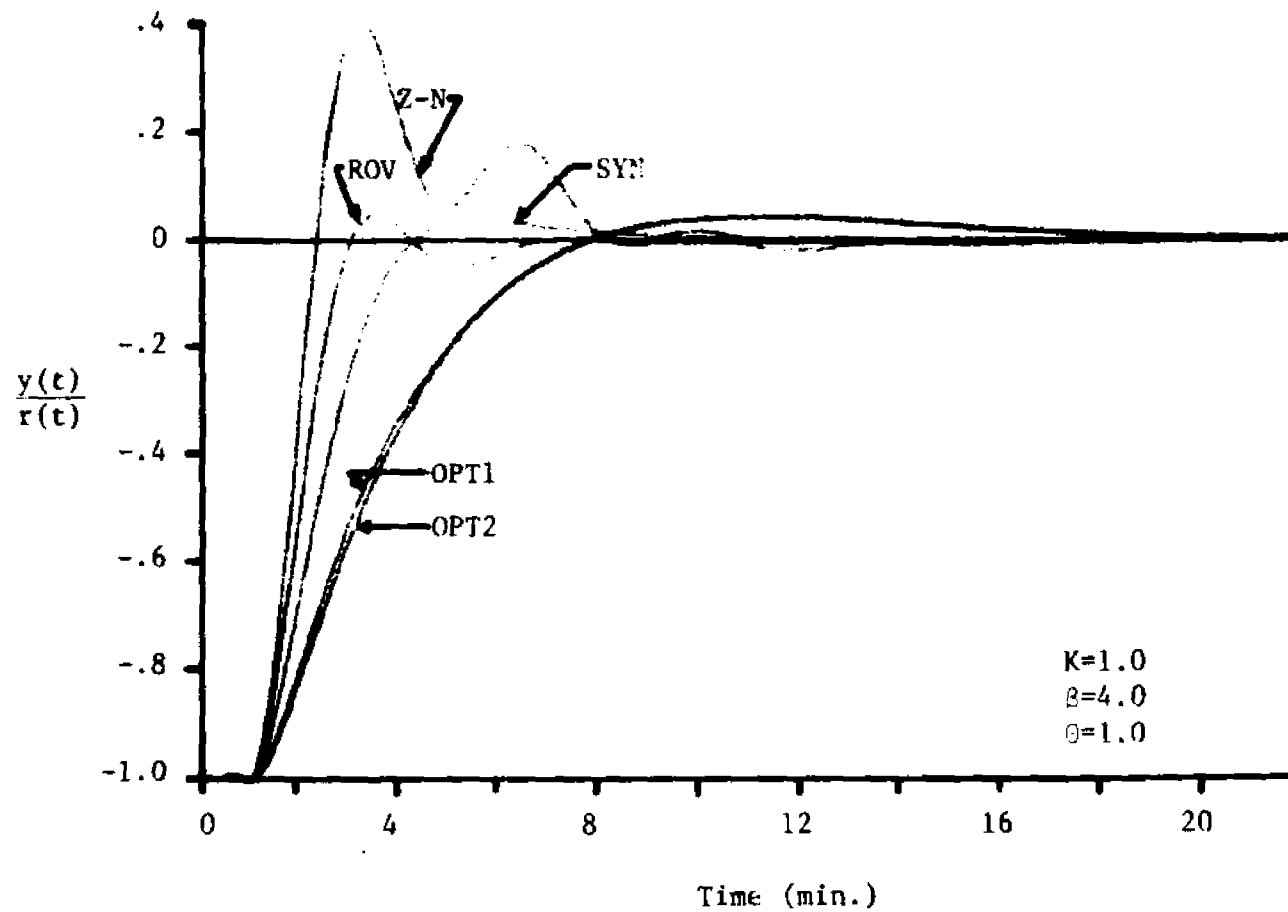
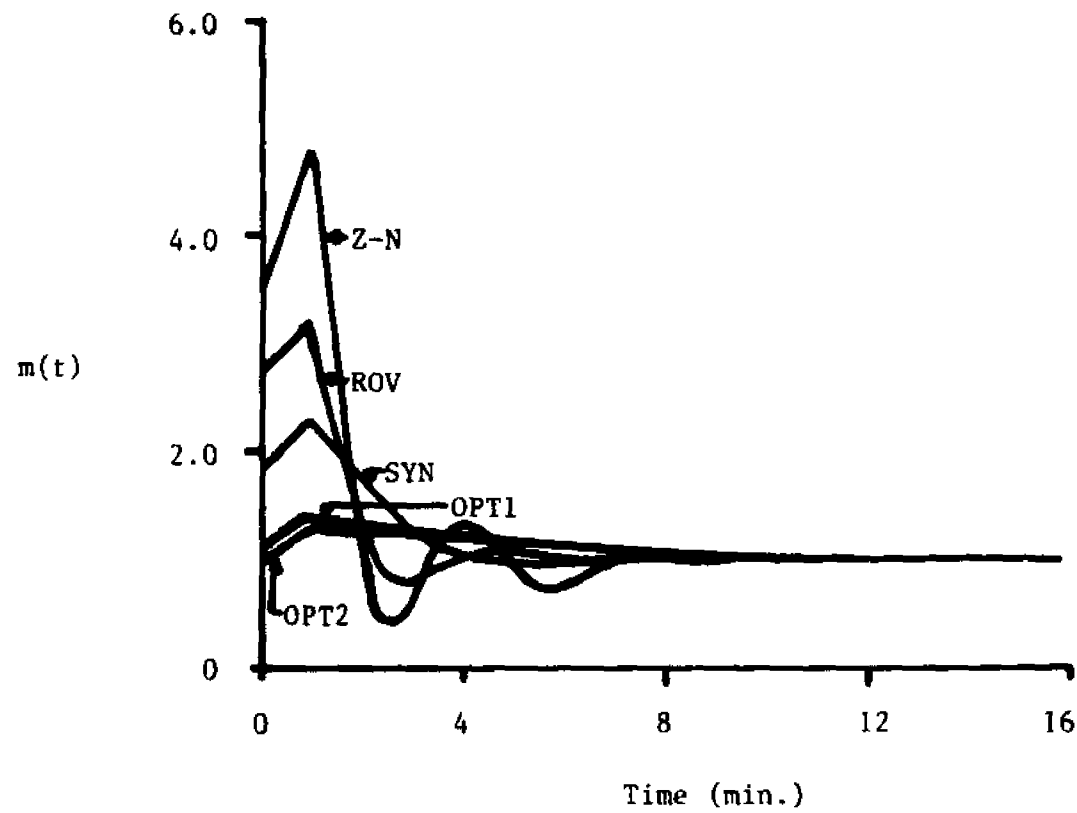


FIGURE 4-32

CONTROLLER OUTPUT OF A SECOND-ORDER OVERDAMPED PROCESS
WITH LONG DEAD-TIME FOR A PID CONTROLLER



approximation for the process dead-time, however, the following equations show that the technique attempts to represent the smaller time constant of a second-order system with an artificial time lag. The equation for the transfer function of a second-order-lag-plus-dead-time process model with the first-order Taylor series expansion for the dead-time is

$$G(s) = \frac{K(1 - t_o s)}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad [4.1]$$

The transfer function of a first-order-plus-dead-time process model with the first-order Pade' approximation for the dead-time is

$$G(s) = \frac{K(1 - \frac{t_o + \Delta}{2} s)}{(\tau s + 1)(\frac{t_o + \Delta}{2} s + 1)} \quad [4.2]$$

Referring back to Chapter III and Figure 3-3, τ and Δ approach τ_1 and τ_2 , respectively, as τ_2/τ_1 approach zero. Therefore, for highly overdamped processes with short dead-time (same order of magnitude as Δ), equations 4.1 and 4.2 are very similar. For these cases, the lag term represents the secondary lag in equation 4.1 and the numerator represents the process dead-time (numerator term in equation 4.1). A process which illustrates this is one which has the parameters of Figure 4-30 (see Table 4-2 for process and model parameters). As one would expect, the PID controller settings resulting from the two models are equivalent and result in identical close-loop responses, Figure 4-30. However, if the second-order process dead-time, t_o , is significantly different from Δ or if the process is not quite so overdamped, the two transfer functions are quite different and result in different controller settings. Examples of the different closed-

loop responses are shown in Figures 4-28, 4-29 and 4-31, and the process and model parameters are presented in Table 4-2. Note, for the critically damped short dead-time case the responses are slightly different, however, increasing the dead-time makes a significant difference in the two responses. The difference in response is due to changes in the reset and preact times to compensate for the artificial lag which is really process dead-time or transportation lag. Since dead-time contributes only to the phase angle shift and introduces no dynamics to the system, the Pade' approximation should be avoided if possible.

Although no plots have been shown, a detailed study of the $PIDD^2$ controller revealed that the $PIDD^2$ controller offered little advantage over the PID controller and in most cases the closed-loop response was slower than the PID closed-loop response. Dropping the D^2 term of $PIDD^2$ controller was also considered to determine the effect of the additional term. This was found to have negligible effect on the closed-loop response and in the typical case the overshoot remained approximately the same or dropped to a value between 4.7% and 5.0% accompanied by a small change in rise time. As expected, the largest changes were experienced when the process approached the underdamped case.

Conclusion

Although the application of the optimal regulator is attractive from a theoretical standpoint it did not perform too well for highly overdamped processes for the criteria selected. A major disadvantage is that the technique does not provide for field adjustment in industrial application. Also, it requires a significant effort to

determine the values of p which will produce the desired responses for a specific system.

It should be realized that if control valve wear is a high maintenance item to the point that some control sluggishness may be tolerated, the optimal regulator may offer a compromising solution.

The controller tuning techniques discussed in Chapters III and IV will be applied to a simulation of a non-linear continuous stirred tank reactor in the next chapter.

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CHAPTER V

APPLICATION OF TUNING TECHNIQUES TO A
NON-LINEAR BACKMIX REACTOR

Introduction

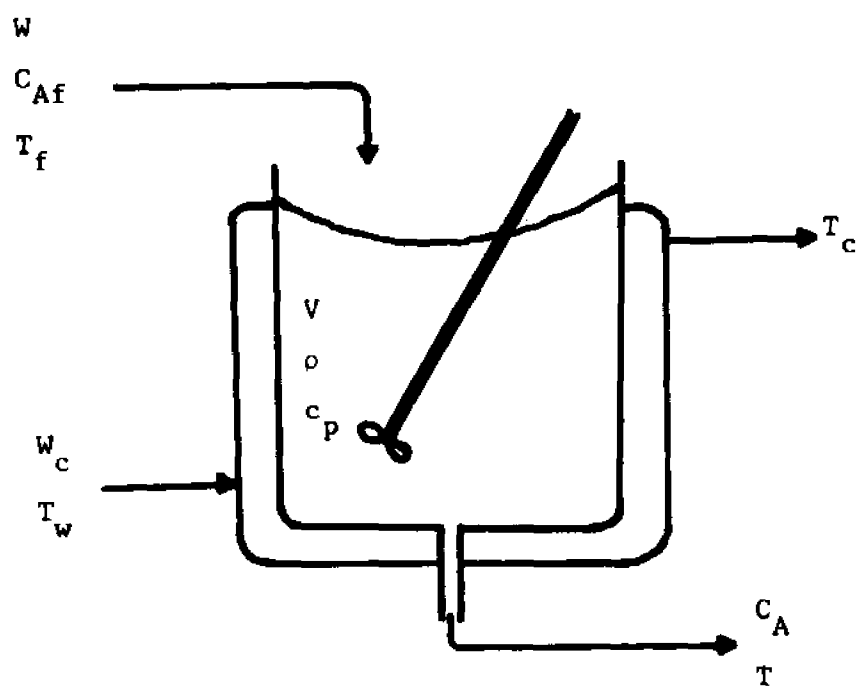
The purpose of this chapter is to apply the controller synthesis and output regulator tuning relationships developed in Chapters III and IV to a non-linear backmix reactor. These results will be compared with the responses obtained by tuning with the following criteria: Rovira (1), Lopez (2), and Ziegler-Nichols (3). Both the PI and PID controllers will be examined for set-point and load or disturbance changes.

Process Description

The process chosen for this comparison is a water-jacketed backmix reactor in which an exothermic second-order reaction is taking place. A schematic diagram of the process is illustrated in Figure 5-1. The contents of the reactor are assumed to be perfectly mixed and the density and specific heat of the reactant and product streams are considered to be constant. Heat losses to the surroundings are considered negligible and the overall heat transfer coefficient, U , is assumed to be constant.

The equations describing this process are given in Table 5-1. Equation 5.1 is an unsteady-state material balance for component A in the reactor. The term $k C_A^2$ is the rate of disappearance of A by chemical reaction. The stoichiometry is given by:

FIGURE 5-1
REACTOR FLOW DIAGRAM



Reaction: $2 A \rightarrow B$

TABLE 5-1
REACTOR EQUATIONS

$$\frac{dC_A}{dt} = \frac{W}{V\rho} (C_{Af} - C_A) - k C_A^2 \quad [5.1]$$

$$V\rho c_p \frac{dT}{dt} = W c_p (T_f - T) - UA' (T - T_c) + (-\Delta H) V k C_A^2 \quad [5.2]$$

$$M_c \frac{dT_c}{dt} = UA' (T - T_c) - W_c (T_c - T_w) \quad [5.3]$$

$$k = k_o \exp \left(-\frac{a}{T} \right) \quad [5.4]$$

$$2A \xrightarrow{k} B \quad [5.5]$$

where k is the reaction rate constant given by the Arrhenius expression, equation 5.4. In this equation T must be in degrees Rankine ($^{\circ}R$), a is the Arrhenius temperature constant ($^{\circ}R$), and k_0 is the Arrhenius rate constant ($\text{ft.}^3/\text{lb-min}$). Equation 5.2 is an unsteady-state energy balance on the reacting mixture. The term $(-\Delta H)$ is the heat of reaction and has the units of Btu/lb of A consumed. Equation 5.3 is an unsteady-state energy balance around the jacket.

Corripio and Smith (4) have simulated this process on analog computer and their analog diagram is presented in Figure 5-2. This diagram with a PI or PID controller has been wired on the EAI 680 analog computer in the Chemical Engineering Departments Hybrid Simulation Laboratory to obtain the needed responses.

The temperature of the reacting mixture, T , will be controlled by manipulating the cooling water rate, W_c , as shown in Figure 5-3. The transfer function of the temperature transmitter and the cooling water flow controller are assumed to have negligible effect on the process.

The process parameters and initial steady state conditions for the backmix reactor are presented in Table 5-2.

Process Characteristics

Although some simplifying assumptions were made in obtaining the process equation, the most important non-linearities of the process are included in the simulation. Corripio and Smith (4) showed that the temperature response for $+5^{\circ}F$ set-point change was different than for $-5^{\circ}F$, a result of nonlinearities. This is illustrated in Figure 5-4, which presents ΔT , the change in tempera-

FIGURE 5-2

ANALOG DIAGRAM FOR CHEMICAL REACTOR

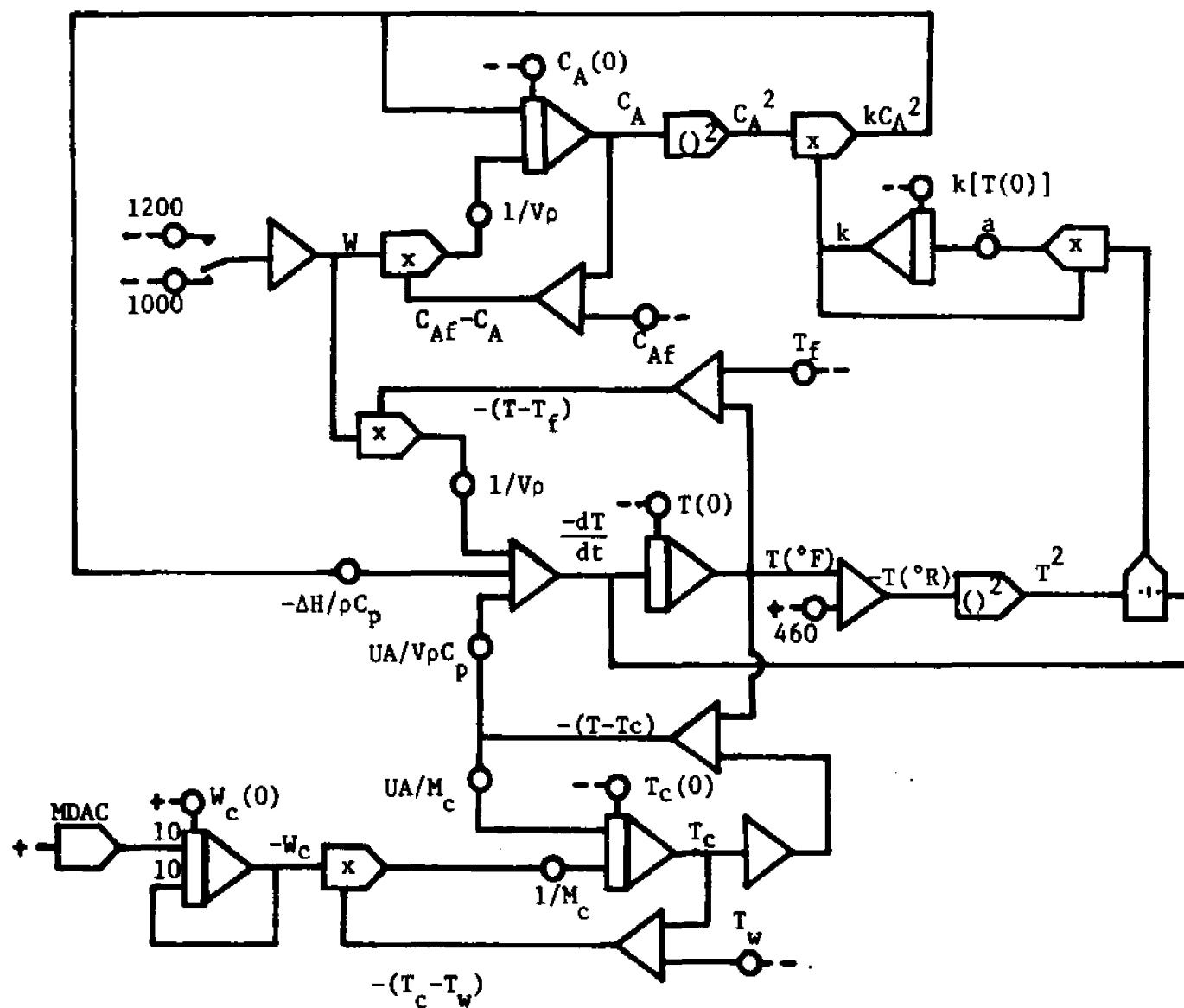


FIGURE 5-3

SCHEMATIC DIAGRAM OF CONTROL SYSTEM FOR CHEMICAL REACTOR

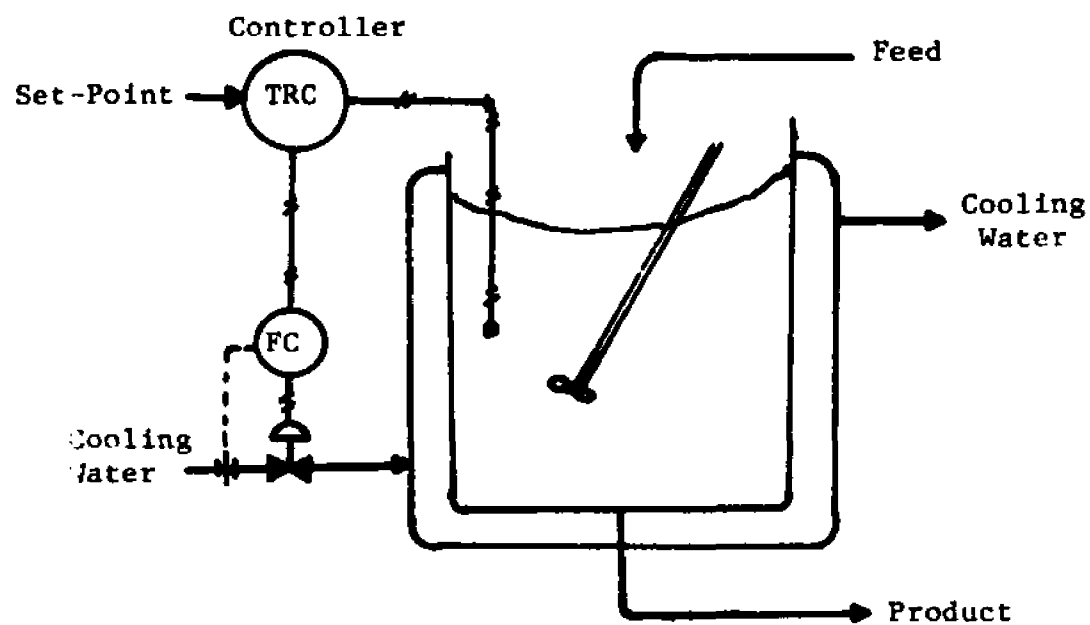


TABLE 5-2

Steady State Conditions

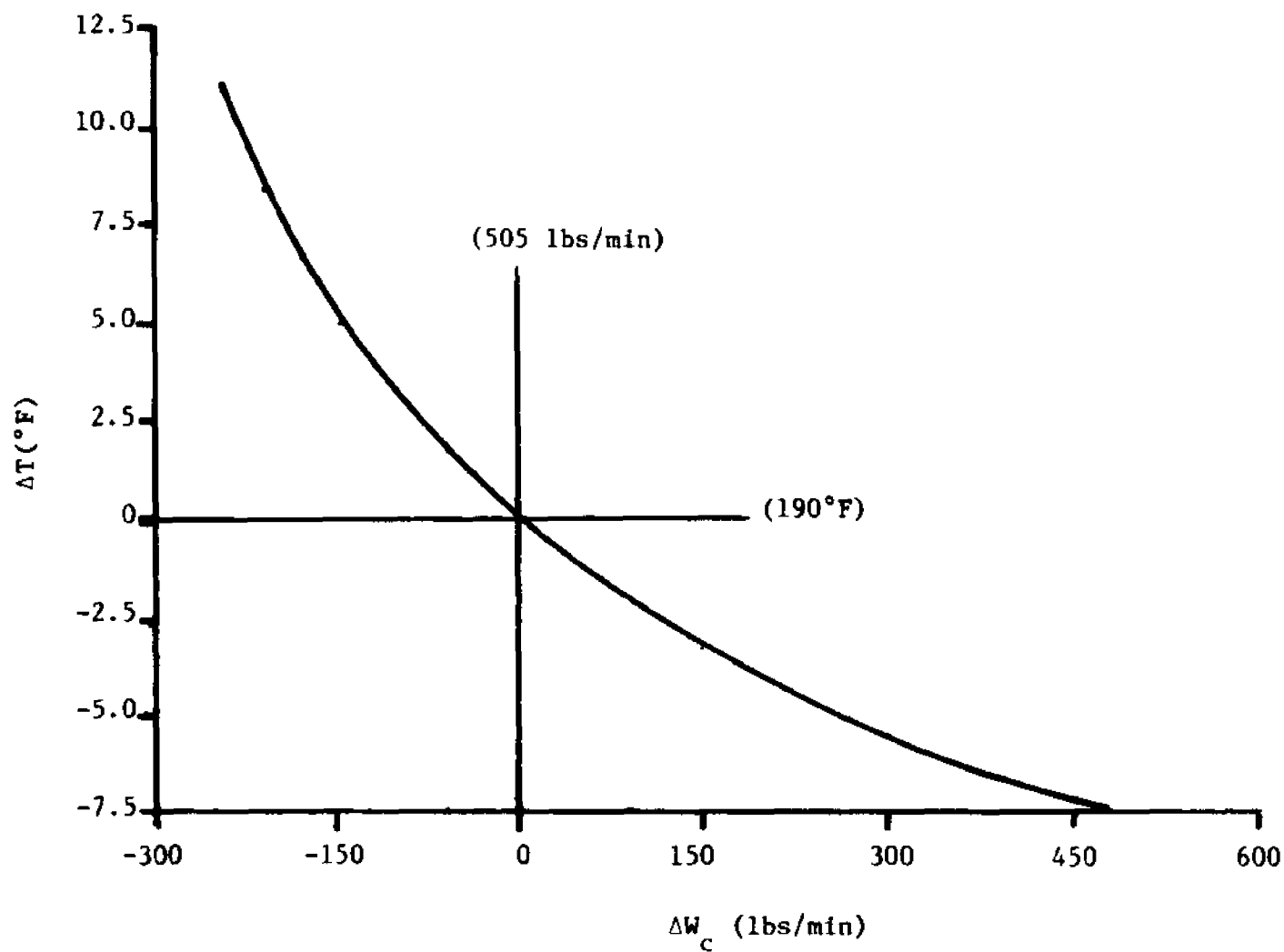
T	$= 190^{\circ}\text{F}$
k	$= 0.0278 \text{ ft}^3/\text{lb-min}$
W	$= 1000 \text{ lb/min}$
T_f	$= 150^{\circ}\text{F}$
T_c	$= 120^{\circ}\text{F}$
C_{Af}	$= 9.0 \text{ lb/ft}^3$
C_A	$= 3.6 \text{ lb/ft}^3$
W_c	$= 505 \text{ lb/min}$
T_w	$= 80.^{\circ}\text{F}$

Systems Parameters

c_p	$= 0.90 \text{ Btu/lb}^{\circ}\text{F}$
V	$= 250 \text{ ft}^3$
ρ	$= 60 \text{ lb/ft}^3$
a	$= 2560^{\circ}\text{R}$
M_c	$= 4,000 \text{ Btu/}^{\circ}\text{F}$
UA'	$= 600 \text{ BTU/min}^{\circ}\text{F}$
$-\Delta H$	$= 867. \text{ Btu/min}^{\circ}\text{F}$
k_o	$= 1.43 \text{ ft}^3/\text{lb-min.}$

FIGURE 5-4

CHANGE IN REACTOR TEMPERATURE AS A FUNCTION
OF THE CHANGE IN COOLING WATER RATE



ture, as a function of ΔW_c , the change in cooling water rate. Note that the slope of this curve is the process gain and that it varies considerably on each side of the operating point.

Model Parameters

A process reaction curve was obtained by introducing a step change in cooling water rate (manipulated variable or controller output) and recording the response of the control variable (transmitter output or controller input). Although the process is nonlinear and different changes in the manipulated variable result in different process reaction curves, only one process reaction curve will be used to determine the model parameters. This would be the typical case in industry, since it is time consuming to obtain a process reaction curve.

The process reaction curve which will be used to determine the model parameters for the backmix reactor is shown in Figure 5-5. The response was obtained for a step change in cooling water rate of -120 lbs/min.

The parameters of the first-order-lag-plus-dead-time model were determined by Miller's (5) technique and are as follows:

$$K \equiv \text{process gain} = -0.0333 \text{ } ^\circ\text{F}/(\text{lbs}/\text{min})$$

$$\tau \equiv \text{process time constant} = 13.95 \text{ min}$$

$$t_o \equiv \text{process dead-time} = 2.5 \text{ min.}$$

The parameters of the second-order-lag-plus-dead-time model were determined by Stern's (6) technique and are as follows:

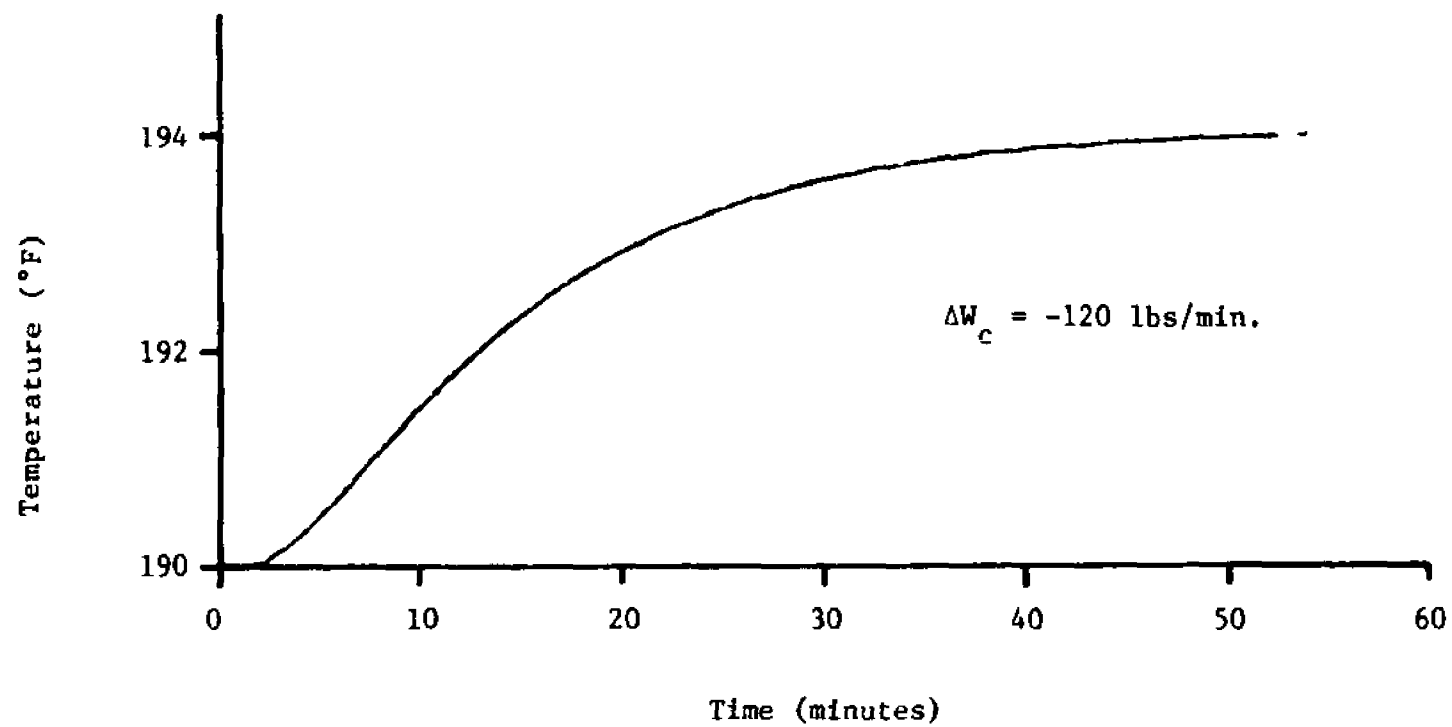
$$K = \text{process gain} = -0.0333 \text{ } ^\circ\text{F}/\text{lbs}/\text{min}$$

$$b \equiv \text{damping parameter} = 0.34 \text{ min}^{-1}$$

$$c \equiv \text{frequency parameter} = 0.024 \text{ min}^{-2}$$

$$t_o \equiv \text{process dead-time} = 0.8 \text{ min.}$$

FIGURE 5-5
PROCESS REACTION CURVE FOR CHEMICAL REACTOR



The time constants for the factored form of the second-order-lag-plus-dead-time model are:

$$\tau_1 = 10.1 \text{ min}$$

$$\tau_2 = 4.1 \text{ min.}$$

Control Scheme

The temperature of the reacting mixture is to be controlled by manipulating the cooling water rate to the jacket. A PI or PID controller has been implemented in the simulation to obtain the closed-loop responses. The above model parameters will be used to determine the PI and PID controller settings for a unit step change in set-point and load (or disturbance) for the following techniques:

1. Ziegler-Nichols (3): quarter decay ratio
- 2a. Rovira (1): IAE for set point changes
- 2b. Lopez (2): IAE for load on disturbance changes
3. Synthesis: 5% overshoot criteria (Chapter III)
4. Output Regulator: 5% overshoot criteria (Chapter IV).

The set-point changes will be a step change in the desired value for the temperature of the reacting mixture of $+4^\circ\text{F}$ and -4°F . The load or disturbance change will be a step-change in the reactant feed rate of $3.2 \text{ ft}^3/\text{min}$ or a 20% increase in reactant concentration.

Results

In the response plots to follow, the different techniques will be denoted as follows:

- Z-N: for Ziegler-Nichols' technique
- ROV: for Rovira's technique
- LOP: for Lopez's technique

- SYN: for controller synthesis 5% overshoot criteria
- OPT1: for output regulator 5% overshoot criteria based on
 the first-order-lag-plus-dead-time model
- OPT2: for output regulator 5% overshoot criteria based on
 the second-order-lag-plus-dead-time model.

The closed-loop responses for the backmix reactor with a PI controller are presented in Figures 5-6 and 5-7 for set-point changes and Figure 5-8 for a load or disturbance change. The effect of process nonlinearities on the closed-loop response is illustrated by the different responses obtained for each technique for positive and negative set-point changes.

The results for set-point changes, Figures 5-6 and 5-7, show that Ziegler-Nichols has the fastest rise time but also the largest overshoot. The closed-loop response is well behaved for a negative set-point change with a 28% overshoot but is oscillatory for a positive set-point change and has a 80% overshoot. The results for Rovira's IAE and controller synthesis 5% overshoot criteria are very similar to each other. The closed-loop responses are slightly oscillatory for a positive set-point change overshooting 34% and 28% respectively and have very good responses for a negative set-point change, overshooting 4% and 2%, respectively. The output regulator responses are somewhat slower but introduces less dynamics into the closed-loop response and overshoots 9% for a positive set-point change and 0% for a negative set-point change.

The closed-loop responses for a load change are presented in Figure 5-8 for the backmix reactor with a PI controller. Since Lopez's relationships were developed for load changes, his

FIGURE 5-6

REACTOR CLOSED-LOOP RESPONSE WITH A PI CONTROLLER FOR POSITIVE SET-POINT CHANGE

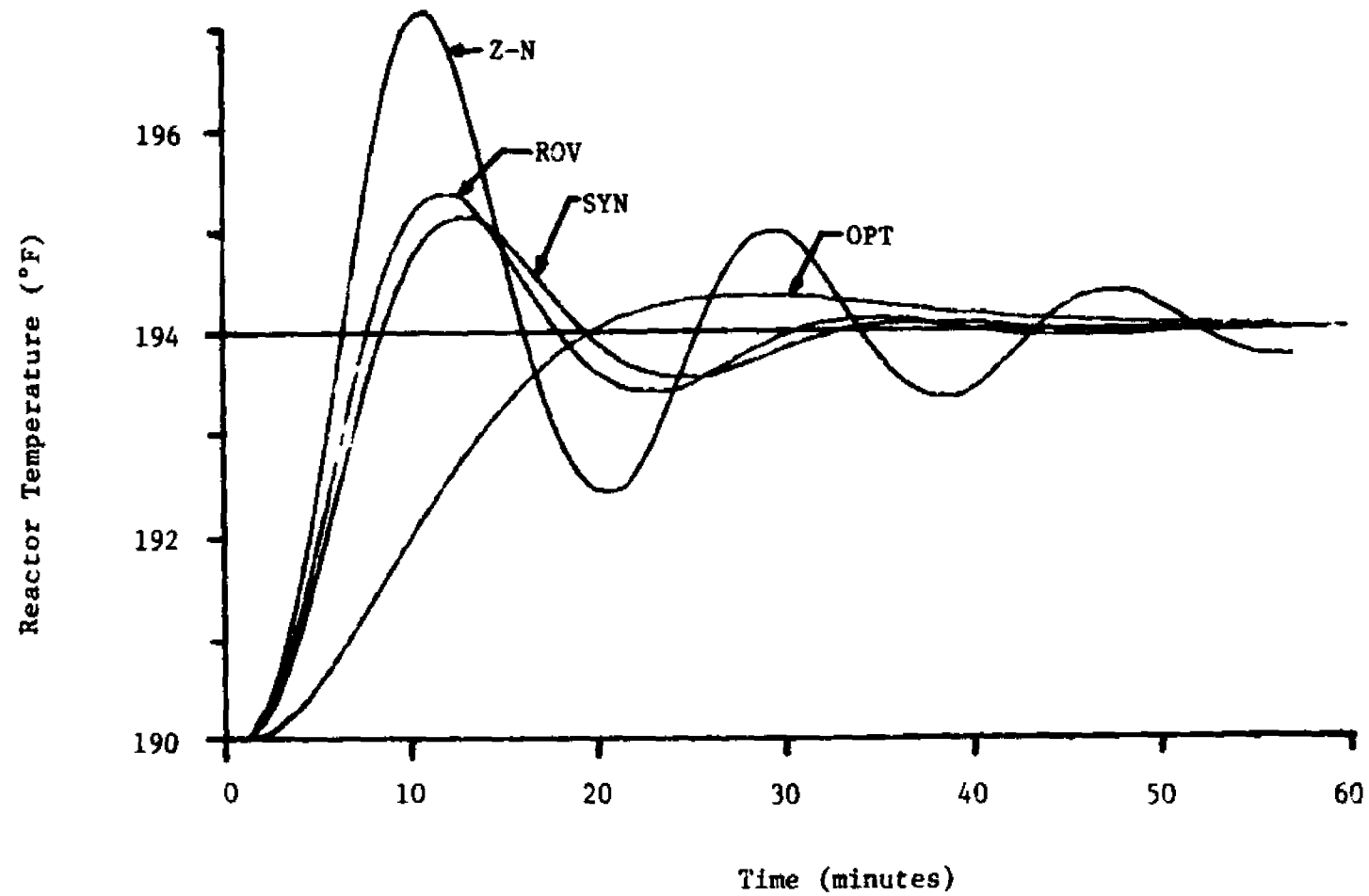


FIGURE 5-7

REACTOR CLOSED-LOOP RESPONSE WITH A PI CONTROLLER

FOR NEGATIVE SET-POINT CHANGE

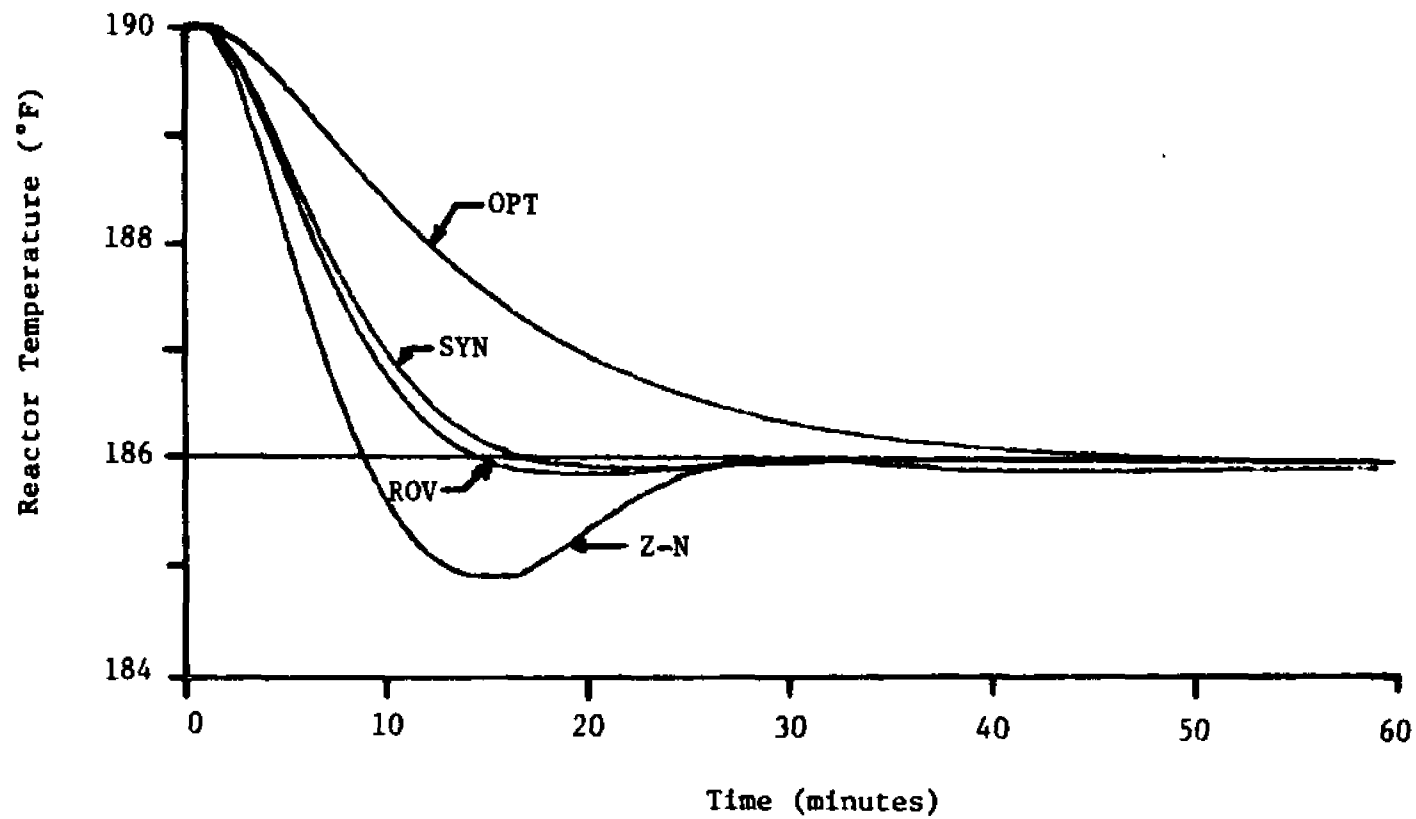
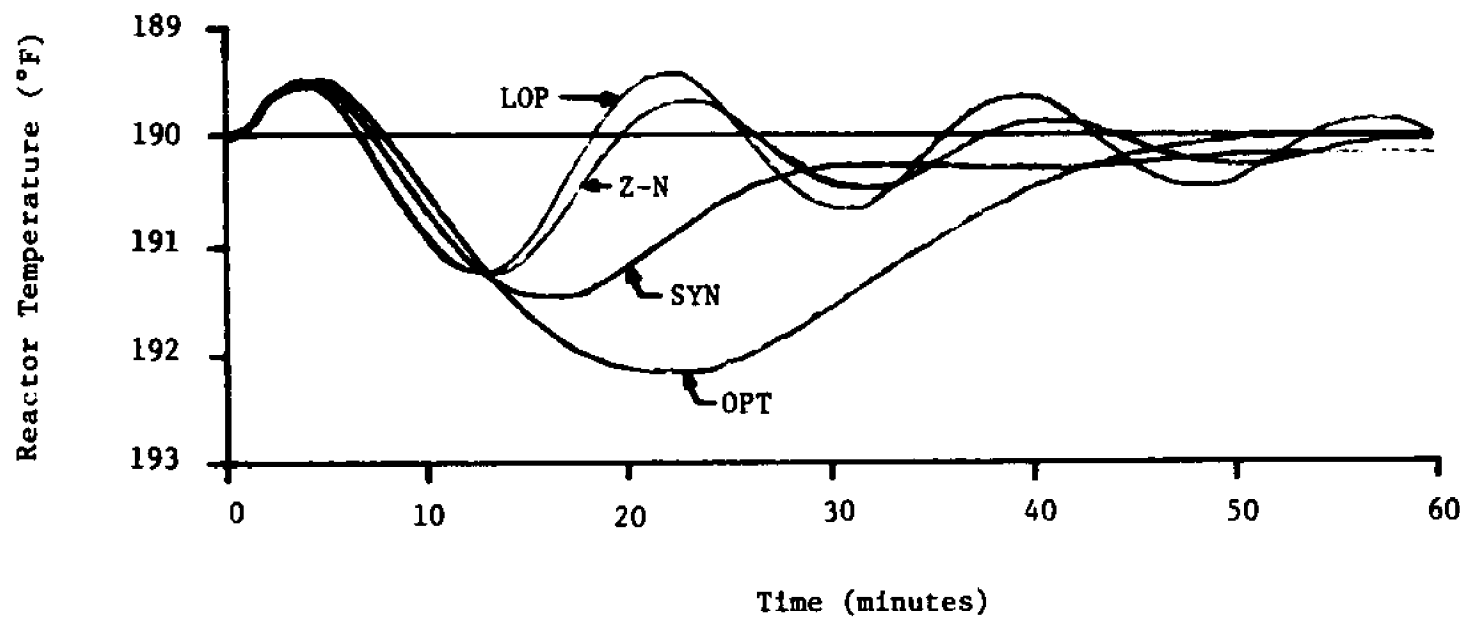


FIGURE 5-8
REACTOR CLOSED-LOOP RESPONSE WITH A PI CONTROLLER
FOR A 20% LOAD CHANGE



tuning relationships will be used in place of Rovira's. The responses, shown in Figure 5-8, indicate that Ziegler-Nichols and Lopez's tuning result in an oscillatory system behavior while controller synthesis and the output regulator tuning are well behaved. Although the output regulator tuning is well behaved, dynamically, it is rather slow and also allows the controlled variable (reactor temperature) to stray farthest from its desired value. The values of the tuning parameters for the PI controller for these plots are presented in Table 5-3.

The closed-loop responses for the backmix reactor with a PID controller are presented in Figures 5-9 and 5-10 for set-point changes and Figure 5-11 for load change. The values of the tuning parameters for the PID controller for these plots are presented in Table 5-4. As illustrated by Figures 5-9 and 5-10, Ziegler-Nichols tuning again results in the largest overshoots, 78% and 42% for positive and negative set-point changes respectively.

The responses for IAE tuning of the PID controller are better than the responses for IAE tuning of the PI controller. For a positive set-point change the overshoot is 20% and for a negative set-point change the overshoot is 0%. The responses of the controller-synthesis technique and both first-order and second order output regulators are very similar with the output regulator based on the second order model being the slowest. The overshoot for a positive set-point change for all three methods was 8% and was 0% for a negative set-point change. Rovira's technique offers the best response for these cases.

As shown in Figure 5-11, Ziegler-Nichols results in a very

FIGURE 5-0

REACTOR CLOSED-LOOP RESPONSE WITH A PID CONTROLLER FOR A POSITIVE SET-POINT CHANGE

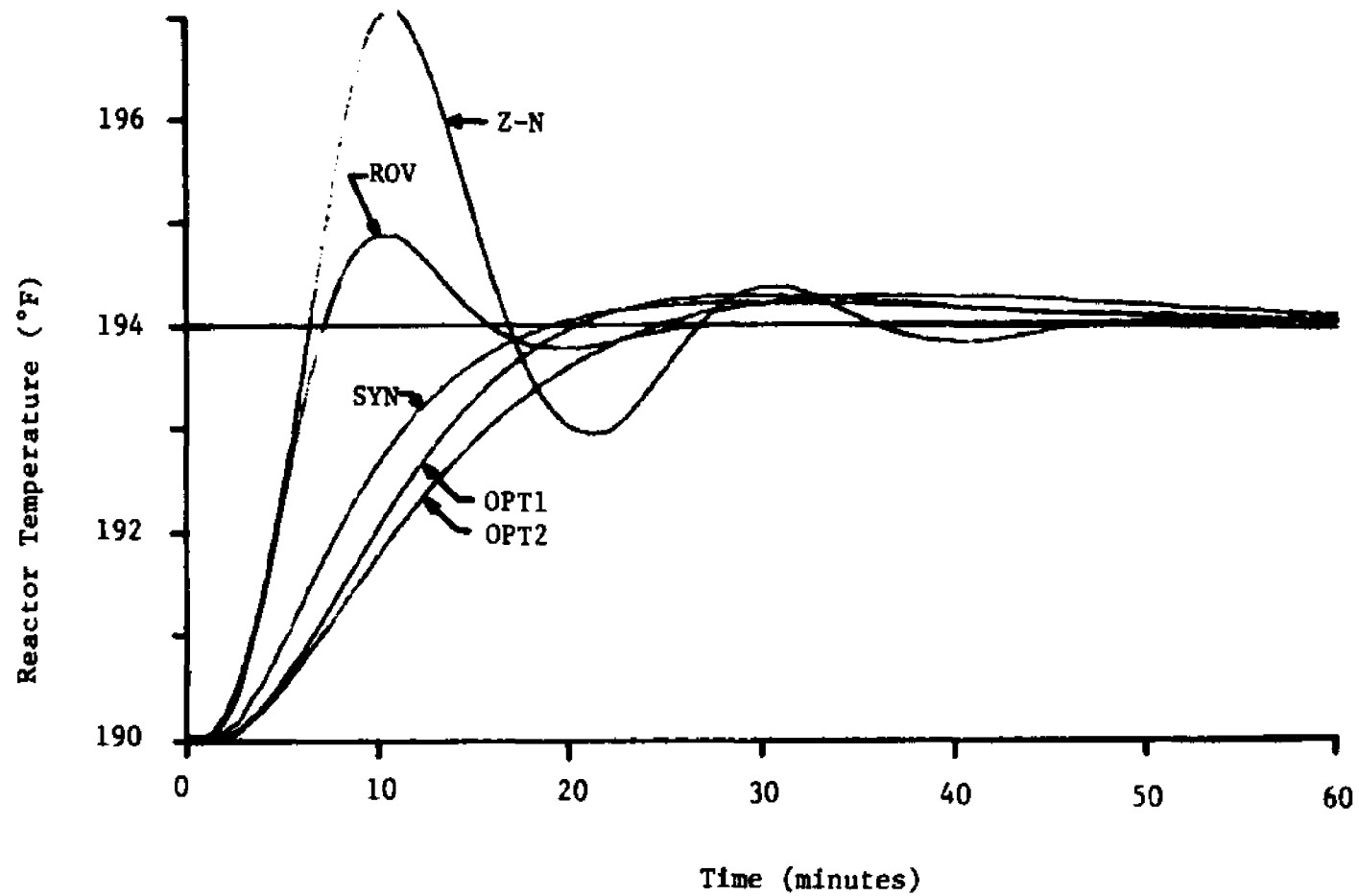


FIGURE 5-10

REACTOR CLOSED-LOOP RESPONSE WITH A PID CONTROLLER FOR A NEGATIVE SET POINT CHANGE

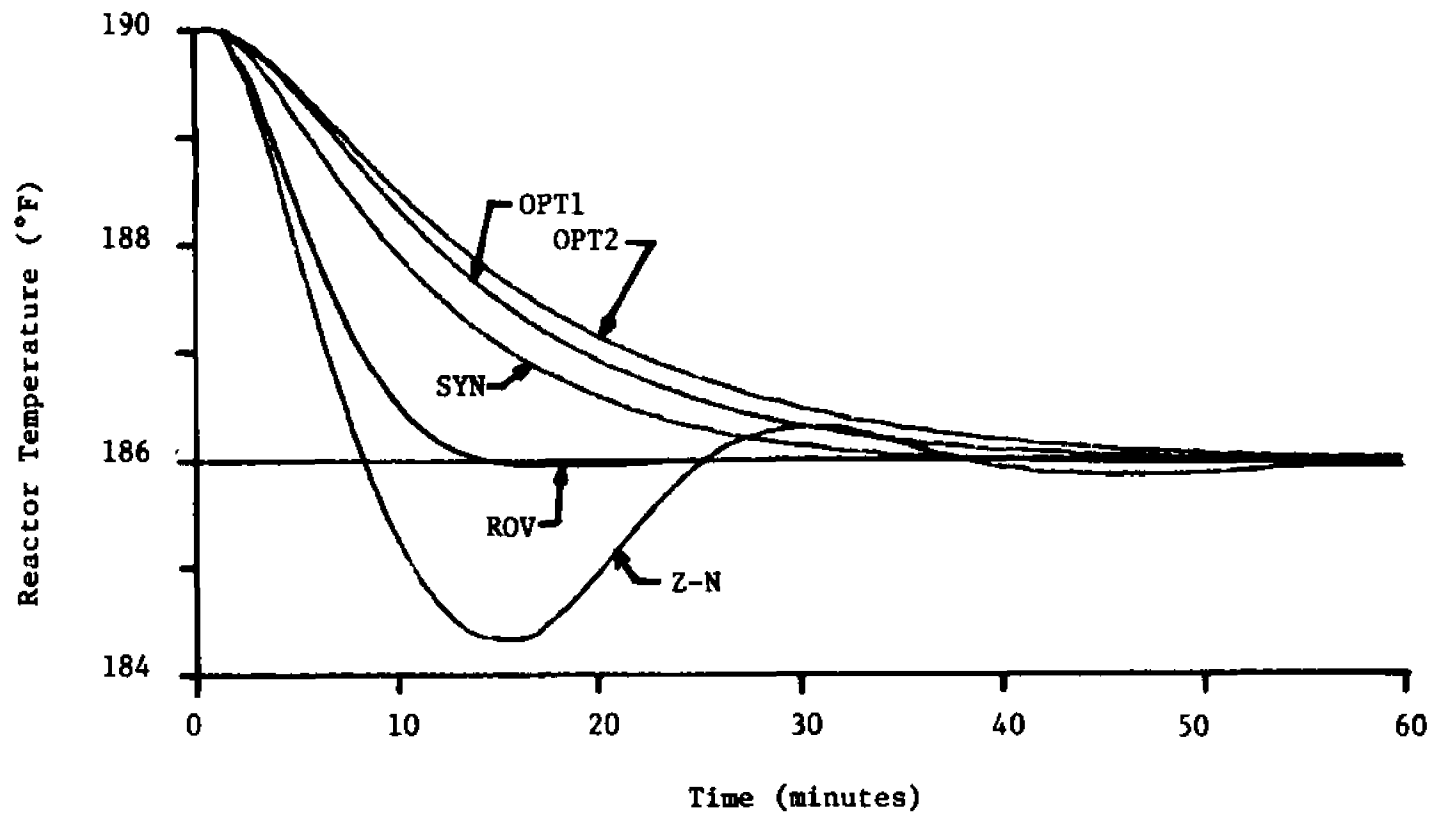


FIGURE 5-11

REACTOR CLOSED-LOOP RESPONSE WITH A PID CONTROLLER FOR A LOAD CHANGE

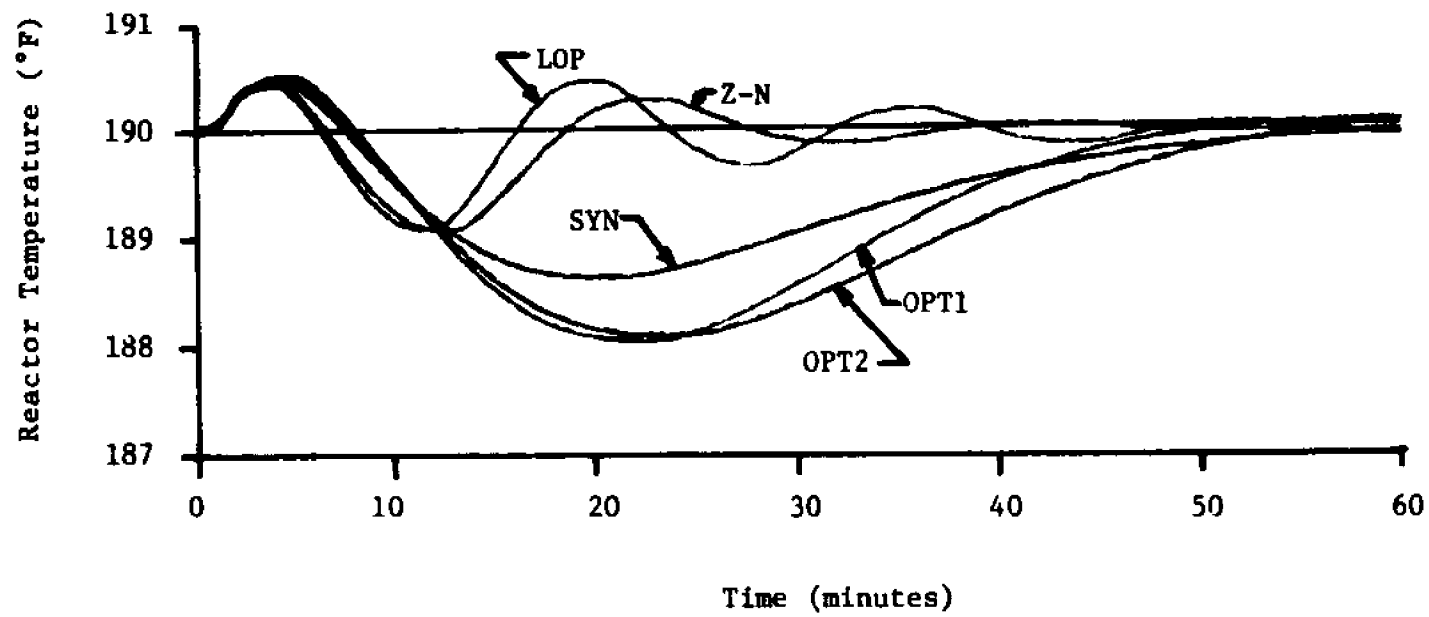


TABLE 5-3

PI CONTROLLER TUNING PARAMETERS - BACKMIX REACTOR

Techniques	K_c (lbs/min/°F)	T_i min
Controller Synthesis (5% overshoot)	-0.584	13.95
Rovira (Set-Point) (IAE)	-0.666	14.50
Lopez (Load) (IAE)	-1.072	6.805
Ziegler-Nichols (quarter decay)	-1.003	8.32
Output Regulator (5% overshoot)	-0.214	9.66

TABLE 5-4
PID CONTROL TUNING PARAMETERS - BACKMIX REACTOR

Techniques	K_c lbs/min/°F	T_i min.	T_d min.
Controller Synthesis (5% overshoot)	-0.404	14.18	2.94
Rovira (Set-Point) (IAE)	-0.968	19.46	1.01
Lopez (Load) (IAE)	-1.398	4.383	0.952
Ziegler-Nichols (quarter-decay)	-1.116	5.00	1.25
Output Regulator (1) (5% overshoot)	-0.241	10.52	1.10
Output Regulator (2) (5% overshoot)	-0.225	10.96	2.65

good response to load change. Lopez's technique is somewhat oscillatory while controller synthesis and the optimal regulators are well behaved but slower to recover. Note, while both optimal regulator responses rather far from the set-point the optimal regulator based on the second-order model recovers fastest.

The ITAE tuning for Lopez and Rovira were also considered but gave similar results to the IAE tuning therefore were not presented in the plots.

Conclusions

The figures just examined serve to illustrate the closed-loop responses obtainable by the tuning relationships considered.

Although controller synthesis did not always result in the best response for the cases considered it illustrated the synthesis tuning is rather stable and showed no tendency to instability in spite of the fact that the process gain varies so much. The synthesis technique can be conceptually tuned in the field by simple adjustment of the gain only and the other techniques are conceptually based on interrelationships between all tuning parameters. This fact provides an attractive feature for industrial application since tuning controllers are a major task in many industries.

REACTOR NOMENCLATURE

A	- Reactants
A'	- Heat transfer area
a	- Arrhenius temperature constant
B	- Products
C_A	- Concentration of A
C_{Af}	- Concentration of A in the reactor feed
c_p	- Heat capacity of the reacting fluid
ΔH	- Heat of reaction
k	- reaction rate constant
k_0	- Arrhenius rate constant
M_c	- Total heat capacity of reacting mass
t	- Time
T	- Temperature of the reacting fluid
T_c	- Temperature of the cooling water leaving the reactor jacket
T_f	- Temperature of the reactor feed
T_w	- Temperature of the cooling water
U	- Heat transfer coefficient
V	- Reactor volume
W	- Mass rate of feed
W_c	- Mass rate of cooling water
ρ	- Density of reacting fluid

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CHAPTER VI

CONCLUSIONS

The purpose of this dissertation is to investigate controller tuning techniques which provide the control engineer some flexibility in selecting a desired response and to examine its feasibility for industrial applications. In this chapter, the conclusions drawn from the results presented in the previous chapters are summarized.

The controller synthesis results for 5% and 1% overshoot criteria was found to produce a highly stable closed-loop response with little tendency to oscillate. It also resulted in a relatively fast response with short rise time and settling times. Comparison of the controller synthesis parameters for 5% overshoot criteria with those of the minimum integral of the absolute value of the error parameters for set-point changes showed close agreement for most of the range of parameter values studied.

Although this work does not attempt to answer the question of what is the best performance criteria, it is important to note that the 5% overshoot criteria chosen, based on the discussion presented in Chapter III, resulted in closed-loop responses very similar to minimum IAE responses.

The optimal output regulator results showed that the second derivative term of the $PIDD^2$ controller contributed very little to the controller performance. In general, controllers designed by the optimal output regulator technique did not perform as well as those

designed by the synthesis technique. In every case, the rise and settling times were longer than for the corresponding parameters using the controller synthesis technique. This is because the optimal regulator is based on a performance function which contains the error-squared and the system has to be penalized heavily (valve movement) to meet a 5% overshoot criteria.

An important result obtained in the optimal output regulator study is that the Pade' approximation for process dead-time should be avoided if possible. The results show that the approximation is good only if the lag introduced by the dead-time approximation is representative of a lower order time constant in the process which has not been compensated for.

Results have been inconclusive with respect to the question of when to use the first derivative term in the controller. The controller synthesis method seems to indicate that the first derivative term is not required when the damping ratio of the second-order process is greater than about 3. The difficulty then arises with regard to the dead-time term, since this term is not compensated in the synthesis technique.

Of the two controller tuning techniques, only controller synthesis seems to be feasible from an industrial application standpoint. It's simplicity, flexibility in tuning, and ease of use offers the control engineer a technique which provides simple guidelines for specifying and tuning a controller and only requires some knowledge as to the magnitude of the dominant poles.

NOMENCLATURE

<u>A</u>	- Process matrix
<u>B</u>	- Process matrix
b	- Second-order process damping parameter
c	- Second-order process frequency parameter
D(s), d(t)	- Disturbance or load input
e	- Error
G(s)	- Process transfer function
<u>H</u>	- Process matrix
I	- Performance function
IAE	- Integral of the absolute value of the error
ISE	- Integral of the square of the error
ITAE	- Integral of the time-weighted absolute value of the error
<u>J</u>	- Matrix of Riccate variables
<u>Je</u>	- Steady-state solution matrix of the Riccate equation
K	- Steady-state gain of the process
K_c	- Controller gain
K_c'	- Off-the shelf controller gain
<u>k</u>	- Gain Matrix
m	- Manipulated variable
P.B.	- Proportional band
p	- Optimal regulator penalty function
<u>Q</u>	- Process matrix
<u>R</u> , r	- Dimensionless penalty function for output regulator

$R(s), r(t)$	- Set-point or desired value of the control variable
s	- Laplace transform variable
T_d, T_{d1}	- Controller preact or derivative time
T_{d2}	- Controller preact or derivative time squared
T_d'	- Off-the-shelf controller preact time
T_i	- Controller reset or integral time
T_i'	- Off-the-shelf controller reset time
t	- Time
t_o	- Process dead-time or transportation lag
U, u	- Dimensionless manipulated variable
X	- Process state vector
Y	- Process output vector
$Y(s), y(t)$	- Transmitter signal or control variable
ζ	- Dimensionless damping parameter
Λ	- Effective dead-time due to a higher ordered process
λ	- Controller synthesis tuning parameter
ω_n	- Natural frequency
τ	- First-order-lag time constant
τ_1, τ_2	- Second-order-lag time constant
θ	- Dimensionless dead-time
ζ	- Damping ratio

APPENDIX A
COMPARISON OF THE CONTROLLER TRANSFER FUNCTION
FOR THE CONTROLLER SYNTHESIS TECHNIQUE AND THE
DEAD-TIME COMPENSATOR TECHNIQUE

a) Controller Synthesis

For controller synthesis, the controller transfer function has been defined as

$$G_c(s) = \frac{M(s)}{E(s)} = \frac{1}{G(s)} \cdot \frac{Y(s)/R(s)}{1 - Y(s)/R(s)} \quad [A-1]$$

where $G(s)$ is the process transfer function and contains dead-time and $Y(s)/R(s)$ is the closed-loop response and will contain the process dead-time.

Equation A-1 can be written as

$$\frac{M(s)}{E(s)} = \frac{1}{G(s)e^{-t_0s}} \cdot \frac{(Y(s)/R(s))e^{-t_0s}}{1 - (Y(s)/R(s))e^{-t_0s}} \quad [A-2]$$

where $G(s)$ and $Y(s)/R(s)$ does not contain the dead-time element.

From this

$$M(s) [1 - (Y(s)/R(s))e^{-t_0s}] = \frac{1}{G(s)} (Y(s)/R(s)) E(s)$$

or

$$M(s) = \frac{Y(s)/R(s)}{G(s)} [E(s) + G(s) \cdot e^{-t_0s} \cdot M(s)] \quad [A-3]$$

Now subtracting $(Y(s)/R(s)) \cdot M(s)$ from both sides of equation A-3

$$M(s) = \frac{Y(s)/R(s)}{G(s)[1 - Y(s)/R(s)]} \cdot [E(s) + G(s) (e^{-t_0 s} - 1)M(s)] \quad [A-4]$$

$$\text{Let } G'_c(s) = \frac{1}{G(s)} \cdot \frac{Y(s)/R(s)}{1 - Y(s)/R(s)}$$

be the controller transfer function, which is defined as though the process does not contain dead-time, then equation A-4 becomes:

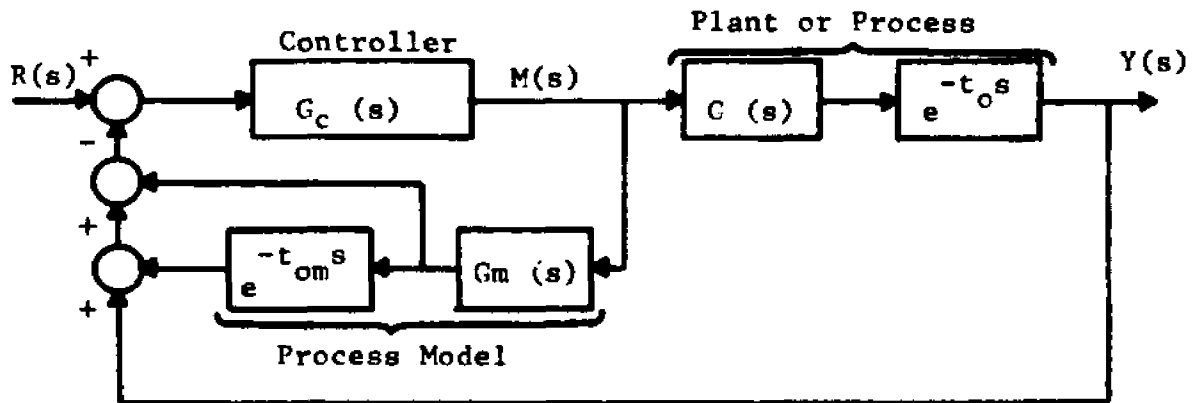
$$M(s) = G'_c(s) [E(s) + G(s) (e^{-t_0 s} - 1) M(s)] \quad [A-5]$$

or

$$G'_c(s) = \frac{M(s)}{E(s) + G(s) (e^{-t_0 s} - 1)M(s)}$$

b) Dead-time Compensator

The block diagram for a dead-time compensator is:



From this diagram

$$M(s) = G_c(s) [E(s) + G_m(s) (e^{-t_{0m} s} - 1)M(s)]$$

or

$$G_c(s) = \frac{M(s)}{E(s) + G_m(s) (e^{-t_{0m} s} - 1)M(s)} \quad [A-6]$$

where $E(s) = R(s) - Y(s)$.

For controller synthesis, the terms $G(s)$ and $e^{-t_0 s}$ are defined by a model of the process, therefore are equivalent to $G_m(s)$ and $e^{t_0 m s}$ in the dead-time compensator. This shows that equations A-5 and A-6 are equivalent for a given process model.

APPENDIX B
TRANSFORMATION OF THE TRANSFER
FUNCTION EQUATIONS TO THE DIMENSIONLESS FORM

a) First-Order-Lag-Plus-Dead-Time Transfer Function

The first-order-lag-plus-dead-time transfer function can be expressed in the Laplace domain as:

$$G(s) = \frac{Y(s)}{M(s)} = \frac{K e^{-t_o s}}{\tau s + 1} \quad [B-1]$$

or in the time domain as

$$\tau \frac{dy}{dt} + y = K m(t - t_o). \quad [B-2]$$

Let σ be the dimensionless time and θ be the dimensionless dead-time and defined by the following expressions:

$$\begin{aligned} \sigma &= t/\tau \\ \theta &= t_o/\tau \end{aligned} \quad [B-3]$$

$$\text{Note, } \frac{dy}{dt} = \frac{dy}{d\sigma} \cdot \frac{d\sigma}{dt} = \frac{1}{\tau} \frac{dy}{d\sigma} \quad [B-4]$$

Substituting equation B-4 into equation B-2 results in the following:

$$\frac{dy}{d\sigma} + y = m(\sigma - \theta) \quad [B-5]$$

which becomes

$$G(s) = \frac{Y(s)}{M(s)} = \frac{K e^{-\theta s}}{s + 1} \quad [B-6]$$

in terms of the dimensionless Laplace transform variable. Equations B-5 and B-6 are dimensionless forms of the first-order-lag-plus-dead-time transfer function.

b) Second-Order-Lag-Plus-Dead-Time Transfer Function

The second-order-lag-plus-dead-time transfer function can be expressed in the Laplace domain as:

$$G(s) = \frac{Y(s)}{M(s)} = \frac{c K_e}{s^2 + bs + c} e^{-t_o s} \quad [B-7]$$

or in the time domain as:

$$\frac{d^2 y}{dt^2} + b \frac{dy}{dt} + cy = c K_m(t - t_o). \quad [B-8]$$

Again let σ denote dimensionless time and θ denote the dimensionless dead-time which are defined by the following expressions:

$$\sigma = \sqrt{c} t \quad [B-9]$$

$$\theta = \sqrt{c} t_o$$

Here

$$\frac{dy}{dt} = \frac{dy}{d\sigma} \cdot \frac{d\sigma}{dt} = \sqrt{c} \frac{dy}{d\sigma} \quad [B-10]$$

and

$$\frac{d^2 y}{dt^2} = \frac{d}{dt} \left(\frac{dy}{dt} \right) = \frac{d}{dt} \left(\sqrt{c} \frac{dy}{d\sigma} \right) = c \frac{d^2 y}{d\sigma^2} \quad [B-11]$$

Substituting equations B-10 and B-11 into equation B-8 results in the following:

$$c \frac{d^2 y}{d\sigma^2} + b \sqrt{c} \frac{dy}{d\sigma} + cy = c K_m (\sigma - \theta) \quad [B-12]$$

$$\text{or } \frac{d^2 y}{d\sigma^2} + \frac{b}{\sqrt{c}} \frac{dy}{d\sigma} + y = K_m (\sigma - \theta). \quad [B-13]$$

Letting

$$\beta = \frac{b}{\sqrt{c}}$$

where β is the damping parameter (dimensionless), equation B-13 can be written in terms of the dimensionless Laplace transform variable as:

$$G(s) = \frac{Y(s)}{M(s)} = \frac{K e^{-\theta s}}{s^2 + \beta s + 1} \quad [B-14]$$

This equation is the dimensionless form of the second-order-lag-plus-dead-time transfer function.

APPENDIX C
TRANSFORMATION OF THE PROCESS MODEL AND
PERFORMANCE FUNCTION EQUATIONS INTO
STATE VARIABLE FORM

1. First-order-lag-plus-dead-time model

The first-order-lag-plus-dead-time model transfer function is given in the dimensionless form by:

$$G(s) = \frac{Y(s)}{M(s)} = \frac{Ke^{-\theta s}}{s + 1} \quad [C-1]$$

where $\theta = t_o/\tau$. This was shown in Appendix B.

a) Taylor series approximation for the dead-time

If the dead-time is approximated with the first-order Taylor series expansion

$$e^{-\theta s} \doteq 1 - \theta s$$

equation C-1 becomes

$$G(s) = \frac{Y(s)}{M(s)} = \frac{K(1 - \theta s)}{s + 1} \quad [C-2]$$

Writing this in the dimensionless time domain the following equation results:

$$\dot{y} + y = Km - K\theta \dot{m} \quad [C-3]$$

Where $\dot{y} = \frac{dy}{d\sigma}$

and $\dot{m} = \frac{dm}{d\sigma}$

To transform this into state variable representation let;

$$X_1 = y, X_2 = Km, \text{ and } u = K\dot{m} \quad [C-4]$$

Then

$$\begin{aligned}\dot{X}_1 &= \dot{y} = -y + Km - K\Theta\dot{m} \\ &= -X_1 + X_2 - \Theta u \\ \dot{X}_2 &= K\dot{m} = u\end{aligned}$$

This can be expressed in matrix form as

$$\dot{\underline{X}} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \underline{X} + \begin{bmatrix} -\Theta \\ 1 \end{bmatrix} u$$

or

[C-5]

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B}u$$

Now since the output y is defined by the state X_1 , the equation defining the output as a function of the states is:

$$y = [1 \quad 0] \underline{X}$$

or, letting $\underline{H} = [1 \quad 0]$,

[C-6]

$$y = \underline{H} \underline{X}$$

Equations C-5 and C-6 are the state variable representation for the first-order-lag-plus-dead-time model of the process with the first-order Taylor series expansion to approximate the dead-time.

b) Pade' approximation for the dead-time

If the dead-time is approximated by the first-order Pade' approximation

$$e^{-\Theta s} \approx \frac{1 - \frac{\Theta}{2}s}{1 + \frac{\Theta}{2}s} = \frac{2 - \Theta s}{2 + \Theta s}$$

equation C-1 becomes

$$G(s) = \frac{Y(s)}{M(s)} = \frac{K(2-\Theta s)}{(s+1)(2+\Theta s)} \quad [C-7]$$

Writing this in the dimensionless time domain results in the

following equation:

$$\ddot{y} + \frac{\Theta+2}{0} \dot{y} + \frac{2}{0} y = \frac{2}{0} K\dot{m} - K\dot{m} \quad [C-8]$$

where $\ddot{y} = \frac{d^2 y}{d\sigma^2}$

$$\dot{y} = \frac{dy}{d\sigma}$$

$$\dot{m} = \frac{dm}{d\sigma}$$

To transform this into state variable representation let:

$$X_1 = y, \quad X_2 = \dot{y} + K\dot{m}, \quad X_3 = K\dot{m}, \quad \text{and } u = K\dot{m} \quad [C-9]$$

Then

$$\dot{X}_1 = \dot{y} = X_2 - X_3$$

$$\dot{X}_2 = \ddot{y} + K\dot{m} = \frac{-2}{0} X_1 - \frac{\Theta+2}{0} X_2 + \frac{\Theta+4}{0} X_3$$

$$\dot{X}_3 = K\dot{m} = u$$

or in matrix form

$$\dot{\underline{X}} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{2}{0} & -\frac{\Theta+2}{0} & \frac{\Theta+4}{0} \\ 0 & 0 & 0 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

or

[C-10]

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} u$$

Again the output y is defined by the state X_1 , so the output

is defined as a function of the states by the following

equation:

$$y = [1 \quad 0 \quad 0] \underline{X}$$

or, letting $\underline{H} = [1 \quad 0 \quad 0]$,

[C-11]

$$y = \underline{H} \underline{X}$$

Equations C-10 and C-11 are the state variable representation for the first-order-lag-plus-dead-time model of the process with the first-order Pade' approximation to approximate the dead-time.

c) Performance function for the first-order-lag-plus-dead-time model.

The quadratic performance function for the time-invariant system is given by:

$$I = \int_0^{\infty} [y^2 + p \dot{m}^2] dt \quad [C-12]$$

Since the model equations are in dimensionless form, it is necessary to transform the performance function to the dimensionless form.

$$\text{Now } \dot{m} = \frac{dm}{dt} = \frac{dm}{d\sigma} \cdot \frac{d\sigma}{dt} = \dot{m}_{\sigma} \frac{d\sigma}{dt}$$

and, since $\sigma = t/\tau$

$$\frac{d\sigma}{dt} = \frac{1}{\tau}$$

$$\text{so } \dot{m} = \frac{1}{\tau} \dot{m}_{\sigma}$$

$$\text{or } \dot{m}^2 = \frac{1}{\tau^2} \dot{m}_{\sigma}^2$$

Equations C-4 and C-9 define u as $K\dot{m}$ where \dot{m} is dimensionless or \dot{m}_{σ} . Substituting this into equation C-12 results in

$$I = \int_0^{\infty} [y^2 + \frac{p}{\tau^2 K^2} u^2] d\sigma$$

or

[C-13]

$$\int_0^{\infty} [y' Q y + u' R u] d\sigma$$

So y , Q , R , and u are scalar quantities and will be denoted by y , q , r , and u . Note r equals $\frac{P}{\tau^2 K^2}$ for the first-order model. Equation C-13 gives the state variable form at the performance function for the first-order-lag-plus-dead-time model of the process.

2. Second-order-lag-plus-dead-time model

The second-order-lag-plus-dead-time model transfer function is given in the dimensionless form by:

$$G(s) = \frac{y(s)}{M(s)} = \frac{K e^{-\theta s}}{s^2 + \beta s + 1} \quad [C-14]$$

where $\theta = t_o \sqrt{c}$ and $\beta = b/\sqrt{c}$

This was shown in Appendix B.

a) Taylor series approximation for the dead-time

If the dead-time is replaced by the first-order Taylor series expansion

$$e^{-\theta s} \doteq 1 - \theta s$$

equation C-14 becomes

$$G(s) = \frac{y(s)}{M(s)} = \frac{K(1-\theta s)}{s^2 + \beta s + 1}$$

Writing this in the dimensionless time domain results in the following:

$$y + \beta \dot{y} + y = Km - \theta K\dot{m} \quad [C-15]$$

To transform this into state variable representation let

$$X_1 = y, X_2 = \dot{y} + \theta K\dot{m}, X_3 = Km, \text{ and } u = K\dot{m} \quad [C-16]$$

Then

$$\begin{aligned} \dot{X}_1 &= \dot{y} = X_2 - \theta X_3 \\ \dot{X}_2 &= \ddot{y} + \theta K\ddot{m} = -X_1 - \beta X_2 + (1 + \theta \beta) X_3 \\ X_3 &= K\dot{m} = u \end{aligned}$$

or in matrix form

$$\dot{\underline{X}} = \begin{bmatrix} 0 & 1 & -\theta \\ -1 & -\beta & (1+\theta\beta) \\ 0 & 0 & 0 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad [C-17]$$

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} u$$

Again the output y is defined by the state X_1 , so the output is defined as a function of the states by:

$$y = [1 \quad 0 \quad 0] \underline{X}$$

or, letting $\underline{H} = [1 \quad 0 \quad 0]$,

$$y = \underline{H} \underline{X}$$

Equations C-17 and C-18 are the state variable representation of the second-order-lag-plus-dead-time model of the process with the first-order Taylor series expansion to approximate the dead-time.

b) Pade' approximation for the dead-time

If the dead-time is approximated by the first order Pade' approximation

$$e^{-\theta s} \approx \frac{2 - \theta s}{2 + \theta s}$$

equation C-14 becomes

$$G(s) = \frac{y(s)}{M(s)} = \frac{K(2-\theta s)}{(s^2 + \beta s + 1)(2 + \theta s)} \quad [C-19]$$

or in the dimensionless time domain is

$$\ddot{y} + \left(\frac{2}{\theta} + \beta\right) \dot{y} + \left(\frac{2}{\theta} \beta + 1\right) y + \frac{2}{\theta} y = \frac{2}{\theta} K_m - K_m \dot{u} \quad [C-20]$$

To transform this into state variable representation let

$$X_1 = v, \quad X_2 = \dot{y}, \quad X_3 = \ddot{y} + K_m, \quad X_4 = K_m, \quad \text{and } u = K_m \dot{u} \quad [C-21]$$

Then

$$\dot{x}_1 = \dot{y} = x_2$$

$$\dot{x}_2 = \ddot{y} = x_3 - x_4$$

$$\dot{x}_3 = \ddot{y} + K\dot{m} = -\left(\frac{2}{\theta} + \beta\right)\ddot{y} - \left(\frac{2}{\theta}\beta + 1\right)\dot{y} - \frac{2}{\theta}y + \frac{2}{\theta}K\dot{m}$$

$$= -\frac{2}{\theta}x_1 - \left(\frac{2}{\theta}\beta + 1\right)x_2 - \left(\frac{2}{\theta} + \beta\right)x_3 + \left(\frac{4}{\theta} + \beta\right)x_4$$

$$\dot{x}_4 = K\dot{m} = u$$

or in matrix form

$$\dot{\underline{X}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -\frac{2}{\theta} & -\left(\frac{2}{\theta}\beta + 1\right) & -\left(\frac{2}{\theta} + \beta\right) & \left(\frac{4}{\theta} + \beta\right) \\ 0 & 0 & 0 & 0 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

or

[C-22]

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} u$$

Again the output y is defined by the state x_1 , so the output is defined as a function of the states by the following equation:

$$y = [1 \quad 0 \quad 0 \quad 0] \underline{X}$$

or, letting $\underline{H} = [1 \quad 0 \quad 0 \quad 0]$,

[C-23]

$$y = \underline{H} \underline{X}$$

Equations C-22 and C-23 are the state variable representation for the second-order-lag-plus-dead-time model of the process with the first-order Pade' approximation to approximate the dead-time.

c) Performance function for the second-order-lag-plus-dead-time model

The quadratic performance function for the time-invariant system was given by equation C-12 or

$$I = \int_0^{\infty} [y^2 + p \dot{m}^2] dt \quad [C-12]$$

Since the model equations are in dimensionless form, it is necessary to transform the performance function to dimensionless form.

$$\text{Now } \dot{m} = \frac{dm}{dt} = \frac{dm}{d\sigma} \cdot \frac{d\sigma}{dt} = \dot{m}_{\sigma} \frac{d\sigma}{dt}$$

and since $\sigma = t/c$

$$\frac{d\sigma}{dt} = 1/c$$

$$\text{or } \dot{m}^2 = c \dot{m}_{\sigma}^2$$

Equations C-16 and C-21 define u as $K\dot{m}$ where \dot{m} is dimensionless or \dot{m}_{σ} .

Substituting this into equation C-12 results in:

$$I = \int_0^{\infty} [y^2 + \frac{pc}{K^2} u] d\sigma$$

or

[C-24]

$$I = \int_0^{\infty} [y' Q y + R u] d\sigma$$

Again Y , Q , R and u are scalar quantities and will be denoted by y , q , r , and u . Note, r equals $\frac{pc}{K^2}$ for the second-order model. Equation C-24 gives the state variable form of the performance function for the second-order-lag-plus-dead-time model of the process.

APPENDIX D
CONTROLLABILITY AND OBSERVABILITY
TEST FOR PROCESS MODELS

A system is controllable if and only if the matrix

$$[\underline{B} \mid \underline{A} \underline{B} \mid \underline{A}^2 \underline{B} \mid \dots \mid \underline{A}^{n-1} \underline{B}]$$

is of rank n , where n is the order of the system. Similarly a system is observable if and only if the matrix

$$[\underline{H}' \mid \underline{A}' \underline{H}' \mid \underline{A}'^2 \underline{H}' \mid \dots \mid \underline{A}'^n \underline{H}']$$

is of rank n , where n is the order of the system. This implies that the above matrices must contain n linear independent column vectors.

1. First-order-lag-plus-dead-time model

a) Taylor expansion for dead-time

$$\underline{A} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}; \quad \underline{B} = \begin{bmatrix} -\theta \\ 1 \end{bmatrix}; \quad \underline{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

controllable?

$$[\underline{B} \mid \underline{A} \underline{B}] = \begin{bmatrix} -\theta & \theta + 1 \\ 1 & 0 \end{bmatrix}$$

rank = 2, therefore controllable.

observable?

$$[\underline{H}' \mid \underline{A}' \underline{H}'] = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

rank = 2, therefore observable.

b) Pade' approximation for dead-time

$$\underline{A} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{2}{0} & -\frac{0+2}{0} & \frac{0+4}{0} \\ 0 & 0 & 0 \end{bmatrix}; \quad \underline{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad \underline{H} = [1 \ 0 \ 0]$$

controllable?

$$[\underline{B} \mid \underline{A} \underline{B} \mid \underline{A}^2 \underline{B}] = \begin{bmatrix} 0 & -1 & \frac{0+4}{0} \\ 0 & \frac{0+4}{0} & -\left(\frac{0+2}{0}\right)\left(\frac{0+4}{0}\right) \\ 1 & 0 & 0 \end{bmatrix}$$

rank = 3, therefore controllable.

observable?

$$[\underline{H}' \mid \underline{A}' \underline{H}' \mid \underline{A}'^2 \underline{H}'] = \begin{bmatrix} 1 & 0 & -\frac{2}{0} \\ 0 & 1 & -\frac{0+2}{0} \\ 0 & -1 & \frac{0+4}{0} \end{bmatrix}$$

rank = 3, therefore observable.

2. Second-order-lag-plus-dead-time model

a) Taylor expansion for dead-time

$$\underline{A} = \begin{bmatrix} 0 & 1 & -0 \\ -1 & -\beta & (1+0\beta) \\ 0 & 0 & 0 \end{bmatrix}; \quad \underline{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad \underline{H} = [1 \ 0 \ 0]$$

controllable?

$$[\underline{B} \mid \underline{A} \underline{B} \mid \underline{A}^2 \underline{B}] = \begin{bmatrix} 0 & -0 & 1+0\beta \\ 0 & 1+0\beta & 0-\beta-0\beta^2 \\ 1 & 0 & 0 \end{bmatrix}$$

rank = 3, therefore controllable.

observable?

$$[\underline{H}' \mid \underline{A}' \underline{H}' \mid \underline{A}'^2 \underline{H}'] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -\beta \\ 0 & 0 & 1+0\beta \end{bmatrix}$$

rank = 3, therefore observable.

b) Pade' approximation for dead-time

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad \underline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

where $a_1 = -\frac{2}{\theta}$, $a_2 = -\left(\frac{2}{\theta} \beta + 1\right)$

$$a_3 = -\left(\frac{2}{\theta} + \beta\right), \quad a_4 = \frac{4}{\theta} + \beta$$

$$\underline{H} = [1 \quad 0 \quad 0 \quad 0]$$

controllable?

$$[\underline{B} \mid \underline{A}\underline{B} \mid \underline{A}^2\underline{B} \mid \underline{A}^3\underline{B}] =$$

$$\begin{bmatrix} 0 & 0 & -1 & a_4 \\ 0 & -1 & a_4 & a_2 + a_3 a_4 \\ 0 & a_4 & a_2 + a_3 a_4 & -a_1 + a_2 a_4 + a_3(a_2 + a_3 a_4) \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

rank = 4, therefore controllable.

observable?

$$[\underline{H}' \mid \underline{A}'\underline{H}' \mid \underline{A}'^2\underline{H}' \mid \underline{A}'^3\underline{H}'] =$$

$$\begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & -1 & a_4 \end{bmatrix}$$

rank = 4, therefore observable.

APPENDIX E
DESIGN OF OPTIMAL OUTPUT REGULATOR
BASED ON SIMPLIFIED MODELS

The solution of the linear output regulator is given by:

$$u = -\underline{R}^{-1} \underline{B}' \underline{J_e} \underline{X} = -\underline{k} \underline{X} = -[k_1 \dots k_n] \underline{X}$$

where \underline{R}^{-1} is the inverse of \underline{R}

\underline{B}' is the transpose of \underline{B}

$\underline{J_e}$ is the steady-state or equilibrium solution to the matrix Riccati equation.

All of the above are defined by the process and the performance function except for the parameter p which must be specified by the designer.

1. First-order-lag-plus-dead-time model

a) Taylor expansion for dead-time

Now

$$-u = [k_1 \ k_2] \underline{X} = k_1 X_1 + k_2 X_2$$

From equation C-4, $u = K\dot{m}$, $X_1 = y$, $X_2 = Km$

$$-u = -K\dot{m} = k_1 X_1 + k_2 X_2$$

$$-K\dot{m} = k_1 y + k_2 Km \quad [E-1]$$

Solving equation C-3 for Km , substituting into equation E-1 and collecting terms result in

$$-(1 + k_2 \tau)K\dot{m} = (k_1 + k_2)y + k_2 \dot{y}$$

Transforming this into dimensionless Laplace domain and solving for $M(s)$ gives

$$M(s) = \frac{-k_2}{K(1 + k_2 \theta)} \left[1 + \frac{k_1 + k_2}{k_2} \cdot \frac{1}{s} \right] Y(s) \quad [E-2]$$

Remember this is in the dimensionless Laplace domain.

Now

$$\begin{aligned} \underline{k} &= R^{-1} \underline{B}' \underline{J} \underline{e} = \frac{1}{r} \underline{B}' \underline{J} \underline{e} \\ &= \frac{1}{r} \begin{bmatrix} -\theta & 1 \end{bmatrix} \begin{bmatrix} J_{e11} & J_{e12} \\ J_{e21} & J_{e22} \end{bmatrix} \\ &= \frac{1}{r} \begin{bmatrix} -\theta J_{e11} + J_{e21} & -\theta J_{e12} + J_{e22} \end{bmatrix} \end{aligned}$$

$$\text{So } k_1 = \frac{1}{r} (-\theta J_{e11} + J_{e21})$$

$$k_2 = \frac{1}{r} (-\theta J_{e12} + J_{e22})$$

Then

$$KK_c = \frac{(-\theta J_{e12} + J_{e22})}{r + \theta(-\theta J_{e12} + J_{e22})} \quad [E-3]$$

$$\frac{1}{T_i} = \frac{-\theta J_{e11} + J_{e21} - \theta J_{e12} + J_{e22}}{-\theta J_{e12} + J_{e22}}$$

b) Pade' approximation for the dead-time

Now

$$-u = [k_1 \quad k_2 \quad k_3] \underline{X} = k_1 X_1 + k_2 X_2 + k_3 X_3$$

From equation C-9 in Appendix C

$$X_1 = y; \quad X_2 = \dot{y} + Km; \quad X_3 = Km; \text{ and } u = K\dot{m}$$

so

$$\begin{aligned} -u &= -K\dot{m} = k_1 X_1 + k_2 X_2 + k_3 X_3 \\ &= k_1 y + k_2 (\dot{y} + Km) + k_3 Km \\ &= K_1 y + k_2 \dot{y} + (k_2 + k_3) Km \end{aligned}$$

Solving equation C-8 for Km, substituting in the above equation

and collecting terms, result in

$$-[1 + \frac{0}{2}(k_2+k_3)]K\dot{m} = k_1 y + k_2 \dot{y} + (k_2+k_3) [\frac{0}{2} \ddot{y} + \frac{0+2}{2} \dot{y} + y]$$

Transforming this equation to the dimensionless Laplace domain and solving for M(s) results in the following:

$$M(s) = - \frac{(0+4) k_2 + (0+2)k_3}{K(0k_2 + 0k_3 + 2)} \left[1 + \frac{(0+4)k_2 + (0+2)k_3}{0k_2 + 0k_3 + 2} \cdot \frac{1}{s} + \frac{0(k_2 + k_3)}{(0+4)k_2 + (0+2)k_3} \cdot s \right] Y(s) \quad [E-4]$$

Now

$$\begin{aligned} \underline{k} &= R^{-1} \underline{B}' \underline{J}e \\ &= \frac{1}{r} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} J_{e11} & J_{e12} & J_{e13} \\ J_{e21} & J_{e22} & J_{e23} \\ J_{e31} & J_{e32} & J_{e33} \end{bmatrix} \\ &= \frac{1}{r} \begin{bmatrix} J_{e31} & J_{e32} & J_{e33} \end{bmatrix} \end{aligned}$$

So

$$k_1 = \frac{1}{r} J_{e31}$$

$$k_2 = \frac{1}{r} J_{e32}$$

$$k_3 = \frac{1}{r} J_{e33}$$

Then

$$KKc = \frac{(0+4) J_{e32} + (0+2) J_{e33}}{0 J_{e32} + 0 J_{e33} + 2 r}$$

$$\frac{\tau}{T_i} = \frac{2 (J_{e31} + J_{e32} + J_{e33})}{(\theta+4) J_{e32} + (\theta+2) J_{e33}} \quad [E-5]$$

$$\frac{T_d}{\tau} = \frac{\theta(J_{e32} + J_{e33})}{(\theta+4) J_{e32} + (\theta+2) J_{e33}}$$

2. Second-order-lag-plus-dead-time model

a) Taylor expansion for dead-time

Now

$$-u = [k_1 \quad k_2 \quad k_3] \underline{X} = k_1 X_1 + k_2 X_2 + k_3 X_3 \quad [E-6]$$

From equation C-16 of Appendix C

$$X_1 = y, \quad X_2 = \dot{y} + \theta K_m, \quad X_3 = K_m \quad \text{and} \quad u = K\dot{m}$$

So

$$\begin{aligned} -u &= -K\dot{m} = k_1 X_1 + k_2 X_2 + k_3 X_3 \\ &= k_1 y + k_2 (\dot{y} + \theta K_m) + k_3 K_m \end{aligned} \quad [E-7]$$

Solving equation C-15 for K_m , substituting into equation E-7 and collecting terms result in

$$\begin{aligned} -[1 + \theta (\theta k_2 + k_3)] K\dot{m} &= [k_2 + \beta(\theta k_2 + k_3)] \dot{y} + \\ &+ (k_1 + \theta k_2 + k_3) y + (\theta k_2 + k_3) \ddot{y} \end{aligned}$$

Transforming this into the Laplace domain and solving for $M(s)$ results in the following:

$$\begin{aligned} M(s) &= - \frac{k_2 + \beta(\theta k_2 + k_3)}{K[1 + \theta (\theta k_2 + k_3)]} \left[1 + \right. \\ &\quad \left. \frac{k_1 + \theta k_2 + k_3}{k_2 + \beta(\theta k_2 + k_3)} \cdot \frac{1}{s} + \frac{\theta k_2 + k_3}{k_2 + \beta(\theta k_2 + k_3)} \cdot s \right] Y(s) \end{aligned} \quad [E-8]$$

Now

$$\begin{aligned}
 \underline{k} &= R^{-1} \underline{B}' \underline{J_e} \\
 &= \frac{1}{r} [0 \quad 0 \quad 1] \begin{bmatrix} J_{e11} & J_{e12} & J_{e13} \\ J_{e21} & J_{e22} & J_{e23} \\ J_{e31} & J_{e32} & J_{e33} \end{bmatrix} \\
 &= \frac{1}{r} [J_{e31} \quad J_{e32} \quad J_{e33}]
 \end{aligned}$$

So

$$k_1 = \frac{1}{r} J_{e31}$$

$$k_2 = \frac{1}{r} J_{e32}$$

$$k_3 = \frac{1}{r} J_{e33}$$

Then

$$KK_c = \frac{(1 + \beta\theta) J_{e32} + \beta J_{e33}}{r + \theta^2 J_{e32} + \theta J_{e33}}$$

[E-9]

$$\frac{1}{\sqrt{c} T_i} = \frac{J_{e31} + \theta J_{e32} + J_{e33}}{(1 + \beta\theta) J_{e32} + \beta J_{e33}}$$

$$\sqrt{c} T_d = \frac{\theta J_{e32} + J_{e33}}{(1 + \beta\theta) J_{e32} + \beta J_{e33}}$$

b) Pade' approximation for the dead-time

Now

$$-u = [k_1 \quad k_2 \quad k_3 \quad k_4] \underline{X} = k_1 X_1 + k_2 X_2 + k_3 X_3 + k_4 X_4$$

From equation C-21 in Appendix C

$$X_1 = y; \quad X_2 = \dot{y}; \quad X_3 = \ddot{y} + Km; \quad X_4 = Km; \quad \text{and } u = K\dot{m}$$

So

$$\begin{aligned}
 -u &= -K\dot{m} = k_1 X_1 + k_2 X_2 + k_3 X_3 + k_4 X_4 \\
 &= k_1 y + k_2 \dot{y} + k_3 (\ddot{y} + Km) + k_4 Km
 \end{aligned}$$

Solving equation C-20 for K_m , substituting into the above equation, and letting

$$f_1 = k_3 + k_4$$

$$f_2 = k_2 + f_1 \left(\beta + \frac{\theta}{2} \right)$$

$$f_3 = k_3 + f_1 \left(1 + \frac{\theta}{2} \beta \right)$$

and $f_4 = 1 + \frac{\theta}{2} f_1$

results in

$$-f_4 K \ddot{m} = f_2 \dot{y} + (k_1 + f_1) y + f_3 \ddot{y} + \frac{\theta}{2} f_1 \dddot{y}$$

Transforming this into Laplace domain and solving for $M(s)$

$$M(s) = \frac{-f_2}{K f_4} \left[1 + \frac{k_1 + f_1}{f_2} \cdot \frac{1}{s} + \frac{f_3}{f_2} s + \frac{\theta}{2} \frac{f_1}{f_2} s^2 \right] Y(s)$$

Now

$$\begin{aligned} \underline{k} &= R^{-1} \underline{B}' \underline{J} e \\ &= \frac{1}{r} [0 \ 0 \ 0 \ 1] \begin{bmatrix} J_{e11} & J_{e12} & J_{e13} & J_{e14} \\ J_{e21} & J_{e22} & J_{e23} & J_{e24} \\ J_{e31} & J_{e32} & J_{e33} & J_{e34} \\ J_{e41} & J_{e42} & J_{e43} & J_{e44} \end{bmatrix} \\ &= \frac{1}{r} [J_{e41} \ J_{e42} \ J_{e43} \ J_{e44}] \end{aligned}$$

So

$$k_1 = \frac{1}{r} J_{e41}$$

$$k_2 = \frac{1}{r} J_{e42}$$

$$k_3 = \frac{1}{r} J_{e43}$$

$$k_4 = \frac{1}{r} J_{e44}$$

Then

$$KKc = \frac{Je_{42} + (\beta + \frac{\theta}{2}) Je_{43} + (\beta + \frac{\theta}{2}) Je_{44}}{r + \frac{\theta}{2} Je_{43} + \frac{\theta}{2} Je_{44}}$$

$$\frac{1}{\sqrt{c} T_1} = \frac{Je_{41} + Je_{43} + Je_{44}}{Je_{42} + (\beta + \frac{\theta}{2}) Je_{43} + (\beta + \frac{\theta}{2}) Je_{44}}$$

$$\sqrt{c} T_{d1} = \frac{(2 + \frac{\theta}{2}\beta) Je_{43} + (1 + \frac{\theta}{2}\beta) Je_{44}}{Je_{42} + (\beta + \frac{\theta}{2}) Je_{43} + (\beta + \frac{\theta}{2}) Je_{44}}$$

$$c T_{d2} = \frac{\frac{\theta}{2} Je_{43} + \frac{\theta}{2} Je_{44}}{Je_{42} + (\beta + \frac{\theta}{2}) Je_{43} + (\beta + \frac{\theta}{2}) Je_{44}}$$

APPENDIX F

Process parameters used to generate data for PI tuning relationships are as follows:

$\beta \quad \theta$	0.2	0.5	0.75	1.0	2.0
2.0	✓	✓	✓	✓	✓
2.2	✓	✓	✓	✓	✓
2.4	✓	✓	✓	✓	✓
2.6	✓	✓	✓	✓	✓
2.8	✓	✓	✓	✓	✓
3.0	✓	✓	✓	✓	✓
3.2	✓	✓	✓	✓	✓
3.5	✓	✓	✓	✓	✓
4.0	✓	✓	✓	✓	✓
5.0	✓	✓	✓	✓	✓
6.0	✓	✓	✓	✓	✓
7.0	✓	✓	✓	✓	✓
8.0	✓	✓	✓	✓	✓
9.0	✓	✓	✓	✓	✓

where: $\theta = t_o \sqrt{c}$

$\beta = b/\sqrt{c}$

The process parameters used to generate data for PID tuning relationships are as follows:

$\theta \backslash \beta$	0.25	.333	0.5	0.75	1.0	2.0
2.0	✓	✓	✓	✓	✓	✓
2.2	✓		✓	✓	✓	✓
2.4	✓		✓	✓	✓	✓
2.6	✓		✓	✓	✓	✓
2.8	✓		✓	✓	✓	✓
3.0	✓	✓	✓	✓	✓	✓
3.2	✓		✓	✓	✓	✓
3.5	✓	✓	✓	✓	✓	✓
4.0	✓	✓	✓	✓	✓	✓
5.0	✓		✓	✓	✓	✓
6.0	✓	✓	✓	✓	✓	✓
7.0	✓	✓	✓	✓	✓	✓
8.0	✓	✓	✓	✓	✓	✓
9.0	✓	✓	✓	✓	✓	✓
12.0	✓	✓	✓	✓	✓	✓

where: $\theta = t_o \sqrt{c}$

$\beta = b \sqrt{c}$

APPENDIX G
COMPUTER
PROGRAM LISTINGS

```

C*****
C*****
C-----CONTROLLER SYNTHESIS PROGRAM LISTING
C*****
C*****
      DIMENSION X(10),X0(10),DX(10),RK1(10),RK2(10),RK3(10),RK4(10)
102 FORMAT(I2)
104 FORMAT(1X,'T0 =',F10.3,/,1X, 6('X0(',I2,')=',E10.4))
105 FORMAT(1X,'RISE TIME = ',F6.3,3X,'SETTLING TIME = ',F6.3,3X,'OVERS
      HOOT = ',F6.4,3X,'IAE = ',F8.4)
100 READ(5,102)NP
      IF(NP .EQ. 0)STOP
      1 J=0
      IOS = 0
      CALL ODE (N,H,T0,X0,M,IP,X,DX,NP,OSHOOT,PCENT)
      IF (N .EQ. 0)GO TO 100
      NP1=N+1
      RTIME = 0.0
      STIME = 0.0
      ESTIME = 0.0
      OSHOOT = 0.0
      DO 13 K=1,M
      T =T0
      DO 5 I=1,N
5      X(I)=X0(I)
      IK=1
      CALL DERIV(T,X,DX,IK)
C      CHECK FOR RISE TIME , SETTLING TIME AND OVERSHOOT
      IF(IOS .GT. 0)GO TO 29
      IF(X(1)-1.0+PCENT)48,28,28
28 ESTIME = T
      IOS = 1
29 GO TO (30,32,38,38),IOS
30 IF ( DX(3) ) 31,31,48

```

```

31 RTIME = T
   IOS = 2
   DX3OLD = DX(3)
   GO TO 48
32 IF ( -X(2) )36,36,34
34 OSHOOT = -DX3OLD
   IF(NP,LT, 99)GO TO 1
   IOS = 3
36 DX3OLD = DX(3)
   GO TO 48
38 Z = ABS(X(1))-1.0)
   IF(Z,GT,PCENT)GO TO 40
   IF(IOS,EQ,4)GO TO 48
   STIME = T
   IOS = 4
   GO TO 48
40 IF(IOS,EQ,3)GO TO 48
   STIME = T
   IOS= 3
48 CONTINUE
   T =T0+(H/2.)
   DO 7 I=1,N
   RK1(I)=DX(I)*H
7   X(I)=X0(I)+(RK1(I)/2.)
   IK=2
   CALL DERIV(T,X,DX,IK)
   DO 9 I=1,N
   RK2(I)=DX(I)*H
9   X(I)=X0(I)+(RK2(I)/2.)
   IK=3
   CALL DERIV(T,X,DX,IK)
   DO 11 I=1,N
   RK3(I)=DX(I)*H

```

```

11  X(I)=XJ(I)+RK3(I)
    T =T0+H
    IK=4
    CALL DERIV(T,X,DX,IK)
    J =J+1
    IF(J.GT.0)GO TO 16
    J =IP
C  WRITE CARD SHOULD BE REPLACED HERE WITHOUT C IN COL 1 FOR OUTPUT
C  WRITE(6,104)T0,(I,X0(I),I=1,N),NP1,X(NP1)
16  T0=T0+H
    DO 13 I=1,N
    RK4(I)=DX(I)*H
13  X0(I)=X0(I)+(RK1(I)+2.*RK2(I)+2.*RK3(I)+RK4(I))/6,
    IF(OSHOOT .EQ. 0,0) OSHOOT =-DX(3)
    IF (NP .LT. 99) GO TO 1
    IF(NP .EQ. 100)NP=8
    IF (OSHOOT .LE. PCENT)STIME=ESTIME
    WRITE(6,105)RTIME,STIME,OSHOOT,X(4)
    GO TO 1
END

```

```

C*****
C-----READ INPUT AND CALCULATE DERIVATIVES
C*****
      SUBROUTINE ODE(N,H,TO,XO,M,IP,X,DX,NP,ORST,PCENT)
      DIMENSION X(10),DX(10),XO(10),XOI(10)
      REAL LAMBDA
501  FORMAT(8F10.0)
502  FORMAT(5I2)
800  FORMAT(/,1X,30(' * '),/,1X,'THE SECOND ORDER PLUS DEAD-TIME PROCE
      1SS PARAMETERS ARE',/,1X,'KP =',F8.3,5X,'BETA =',F8.3,5X,'THETA =',
      2F8.3,/,1X,'THE RESULTS ARE FOR A UNIT CHANGE IN SET POINT',/,1X,30
      3(' * '))
802  FORMAT (1X,'THE CONTROLLER SETTINGS ARE',/,1X,'KC =',E11.4,4X,
      1'TI =',E11.4,4X,'TD1 =',E11.4,4X,'TD2 =',E11.4)
803  FORMAT(/,1X,'THE SECOND ORDER MODEL PARAMETERS ARE THOSE OF THE
      1PROCESS',/,1X,'LAMBDA = ',E11.4)
804  FORMAT(/,1X,'THE FIRST ORDER MODEL PARAMETERS ARE ---- TAU = ',
      1F7.3,3X,' TO = ',F6.3,3X,' DEL = ',F5.3)
806  FORMAT(1X,'LAMBDA = ',E11.4,4X,'TAU*LAMBDA = ',E11.4,4X,'TAU/TO
      1= ',E11.4)
      IF (NP .GT. 9) GO TO 85
      READ(5,502)KASE
      IF(KASE .GT. 0)GO TO 85
      N=0
      RETURN
85  GO TO (101,102,103,101,102,160,105),KASE
C-----INPUT DATA----RUNGE-KUTTA
101  READ(5,502)NI
      READ(5,501)TOI,TMAX,H,PRT
      READ(5,501)(XOI(I),I=1,NI)
C-----INPUT DATA----SYSTEM OR PLANT
      READ(5,501)PK,BETA,THETA
      R = 1.0
      WRITE(6,800)PK,BETA,THETA

```

```

READ(5,501)PCENT
READ(5,502)IMODEL
IF(KASE.EQ. 4)GO TO 150
GO TO 103
102 READ(5,501)PK,BETA,THETA
WRITE(6,800)PK,HETA,THETA
IF(KASE.EQ. 5)GO TO 105
103 CALL CSET(BETA,THETA,CK,TI,TD1,KASE,PCENT)
TD2 = 0.0
WRITE(6,802)CK,TI,TD1,TD2
NP = 99
GO TO 194
105 READ(5,502)IMODEL
150 NP = 10
KASE = 6
J = 1
IF (IMODEL .EQ. 2)GO TO 175
DY = 1.0
TY = 1.0
CALL FOPDTM(TY,DY,BETA,THETAM,BETAM)
DEL = THETAM
THETAM = THETAM + THETA
TAUOTO = BETAM/THETAM
WRITE(6,804)BETAM,THETAM,DEL
LAMBDA = 1.0/THETAM
IF(PCENT.EQ. 0.05)LAMBDA = 1.1/THETAM
IF(PCENT.EQ. 0.01)LAMBDA = 0.79/THETAM
F1 = LAMBDA
GO TO 170
160 IF(J.GT. 1) GO TO 165
J = 2
FACTOR = ABS(ORST-PCENT)
IF(ORST.GT. PCENT)GO TO 161
LAMBDA = (1.0+2.0*FACTOR)*LAMBDA

```

```

GO TO 162
161 LAMBDA = (1.0-2.0*FACTOR)*LAMBDA
162 F2 = LAMBDA
F4 = ORST
GO TO 170
165 F3 = F4
F4 = ORST
IF(ORST .GT. (PCENT*1.01))GO TO 168
IF(ORST .LT. (PCENT*0.99))GO TO 168
NP = 100
IF (IMODEL .EQ. 2)GO TO 166
TAULAM = BETAM*LAMBDA
WRITE(6,806)LAMBDA,TAULAM,TAUOTO
GO TO 167
166 WRITE(6,803)LAMBDA
167 WRITE(6,802)CK,TI,TD1,TD2
GO TO 194
168 LAMBDA = F2-((F2-F1)/(F4-F3))*(F4-PCENT)
IF(LAMBDA .LT. 0.0)LAMBDA = 0.1
F1 = F2
F2 = LAMBDA
170 IF(IMODEL .EQ. 2)GO TO 180
CK = (LAMBDA*BETAM)/(PK*(1.0+LAMBDA*THETAM))
TI = BETAM
TD1 = 0.0
TD2 = 0.0
GO TO 194
175 LAMBDA = 1.0/THETA
F1 = LAMBDA
180 CK = (LAMBDA*BETA)/(PK*(1.0+LAMBDA*THETA))
TI = BETA
TD1 = 1.0/BETA
TD2 = 0.0
194 NDT = THETA/H + 0.49999

```

```

      IP=PRT/H + 0.4999
      N=N1
      T0=T01
      DO 190 I=1,N
190  X0(I)=X0I(I)
      H=(TMAX-T0)/H + 0.4999
      NP1=N+1
      CALL LAG (NDT,INDX)
200  U = CK*(R-X0(1))
      X(NP1)= U
      UNP1=0.0
      RETURN

```

```

C-----
      ENTRY DERIV(T,X,DX,IK)

```

```

C-----

```

```

      GO TO (1,2,2,4),IK
1  U = CK*(R-X(1)+(X(3)/TI)-TD1*X(2)-TD2*(UNP1*PK-BETA*X(2)-X(1)))
      CALL DEDTIM(U,NDT,INDX,UN,UNP1)
      UNK = UN
      X(NP1)= U
      GO TO 5
2  UNK = (UN+UNP1)/2.
      GO TO 5
4  UNK = UNP1
5  YDDOT = PK*(UNK)-BETA*X(2)-X(1)
      DX(1) = X(2)
      DX(2) = YDDOT
      DX(3) = R - X(1)
      DX(4) = ABS(DX(3))
      RETURN
      END

```



```

C*****
C-----TRANSPORTATION LAG OR DEAD TIME SIMULATION
C*****
      SUBROUTINE LAG(N,K)
      DIMENSION A(2000)
      DO 5 I=1,N
5     A(I)=0.0
      I=N
      K=-1
      RETURN
C-----
      ENTRY DEDTIM(X,N,K,XN,XNP1)
C-----
      XN=A(I)
      IP1=I+1
      IF(I.EQ.N)IP1=1
      XNP1=A(IP1)
      A(I)=X
      I=I+1
      IF(I.GT.N)I=1
      K=K+1
      RETURN
      END

```

```

C*****
C-----GET THE CONTROLLER SETTINGS
C*****
      SUBROUTINE CSET(BETA,DTP,CK,TI,TD,KASE,PCENT)
      DIMENSION Y(15)
      IF(KASE .EQ. 3)GO TO 5
      J=1
      KASE = 3
      CALL SET(BETA,DTP,TAU,DT ,Y)
      TOD = TAU/DT
      WRITE(6,501)TOD
501  FORMAT(1X,'TAU/(THETA+DEL) = ',F6.3)
      5  GO TO (10,15,20,25,30,35),J
      10  TLAM = -0.15 + 1.115*TOD
          CK = TLAM/(1.0 + TLAM/TOD)
          TI = TAU
          TD = 0.0
          J = J+1
          PCENT = 0.05
          RETURN
      15  TLAM = -0.20 + 0.8*TOD
          CK = TLAM/(1.0 + TLAM/TOD)
          TI = TAU
          TD = 0.0
          J = J+1
          PCENT = 0.01
          RETURN
      20  CK = Y(6)
          TI = Y(7)
          TD = 0.0
          J = J+1
          PCENT = 0.05
          RETURN

```

```

25 CK = Y(8)
   TI = Y(9)
   TD = Y(10)
      J = J+1
RETURN
30 CK = Y(11)
   TI = Y(12)
   TD = 0.0
      J = J+1
RETURN
35 CK = Y(13)
   TI = Y(14)
   TD = Y(15)
   KASF = 2
RETURN
END

```

```

C*****
C-----CALCULATE THE CONTROLLER SETTINGS
C*****
      SUBROUTINE SET(BETA,DTP,TAU,DT ,Y)
      DIMENSION A(15),R(15),Y(15)
      DATA A/0.984,0.608,1.435,0.878,0.482,0.9,3.33,1.2,2.0,0.5,0.758,
11.02,1.086,0.740,0.348/
      DATA B/-0.986,-0.707,-0.921,-0.749,1.137,-1.0,1.0,-1.0,1.0,1.0,
1-0.861,-0.323,-0.869,-0.130,0.914/
      T=1,
      DY=1.
      CALL FOPDTM(T,DY,BETA,DTM,TAU)
      TAU1=(BETA-SORT(BETA*BETA-4.0))/2.0
      TAU2 = 1.0/TAU1
      WRITE(6,101)BETA,TAU1,TAU2,TAU,DTM
101  FORMAT(////,1X,'B=',F4.1,5X,'TAU1=',F7.4,5X,'TAU2=',F7.4,5X,'TAU=',
1,F7.4,5X,'DEL=',F6.4,/)
      DT=DTM*DTP
      DOT=DT /TAU
      DO 5 I=1,15
5  Y(I)= A(I)*(DOT*B(I))
      Y(2)=Y(2)/TAU
      Y(2)= 1.0/Y(2)
      Y(4)=Y(4)/TAU
      Y(4)= 1.0/Y(4)
      Y(5)=Y(5)*TAU
      Y(7)=Y(7)*TAU
      Y(9)=Y(9)*TAU
      Y(10) = Y(10)*TAU
      Y(12)=A(12)+R(12)*DOT

```

```

Y(12)= Y(12)/TAU
Y(12)= 1.0/Y(12)
Y(14)=A(14)+B(14)*DOT
Y(14)= Y(14)/TAU
Y(14)= 1.0/Y(14)
Y(15)= Y(15)*TAU
      10 WRITE(6,102)PTP,(Y(I),I=6,15)
      102 FORMAT(/,6X,'THETA =',F6.4,/,9X,'ZIEGLER NK',5F10.4,/,9X,'ROVIRA I
      1AE',5F10.4)
      RETURN
      END

```

```

C*****
C-----DETERMINE THE PARAMETERS OF THE FIRST ORDER MODEL
C*****
      SUBROUTINE FOPDTH(T,DY,BETA,THETAM,TAU)
      ZETA = BETA/2.0
      DELY = 0.284*DY
      IF(BETA - 2.0) 1,3,5
      1 C = SORT(1.0-ZETA*ZETA)
      ANG = ATAN2(C,ZETA)
      I = 1
      GO TO 7
      3 I = 2
      GO TO 7
      5 C = SORT(ZETA*ZETA-1.0)
      C4 = ZETA/C
      I = 3
      7 NP = 0
      8 NP = NP+1
      K = 0
      10 K = K+1
      GO TO (15,20,25),I
      15 E = 1.0/(EXP(ZETA*T))
      Y = 1.0-(E/C)*SI-1(C*T*ANG)
      GO TO 40
      20 E = 1.0/(EXP(T))
      Y = 1.0-(1.0+T)*E
      GO TO 40
      25 E=1.0/(EXP((ZETA-C)*T))
      E1=(C+ZETA)*T
      IF(E1.GT. 150.)GO TO 26
      E1 = 1.0/(EXP(E1))
      Y = 1.0-0.5*E*(1.0+C4)-0.5*E1*(1.0-C4)
      GO TO 40

```

```

26 Y = 1.0-0.5*E*(1.0+C4)
40 IF(K ,GT. 1)GO TO 50
   Y2 =Y
   T2 = T
   T = 2.0*T
   GO TO 10
50 YDIFF = ABS(Y-DELY)
   IF(YDIFF ,LE.0.0002) GO TO 60
   Y1 =Y2
   T1 = T2
   Y2 = Y
   T2 = T
   T = T2-(((T2-T1)/(Y2-Y1))*(Y2-DELY))
   GO TO 10
60 IF(NP ,GT. 1) GO TO 70
   TPT03 = T
   DELY = 0.632*DY
   GO TO 8
70 TPT =T
   THETAM = 1.5*TPT03-0.5*TPT
   TAU = 1.5*(TPT-TPT03)
   RETURN
   END

```

```

*****
      INPUT CARDS FOR SYNTHESIS PROGRAM
*****
CARD NO. * VARIABLE * FORMAT * DISCRPTION
*****
      *
      *
1      * NP      *      12      * INDICATOR: MUST BE LESS THAN 10; =0 STOP
      *
2      * KASE     *      12      * TYPE OF CASE TO BE RUN
      *
      *      *      *      * =1: RESULTS FOR Z-N, ROVIRA, SYNTHESIS (1ST ON EQ)
      *      *      *      *      (CARDS A,B,C,D,E,F REQUIRED)
      *      *      *      * =2: RUN THE PREVIOUS CASE OVER BUT CHANGE THE
      *      *      *      *      PROCESS PARAMETERS
      *      *      *      *      (CARD D REQUIRED)
      *      *      *      * =3: USED BY PROGRAM INTERNALLY
      *      *      *      * =4: CONTROLLER SETTINGS DETERMINED BY THE
      *      *      *      *      SYNTHESIS TECHNIQUE BUT SEARCH ON LAMBDA FOR
      *      *      *      *      OVERSHOOT CRITERIA
      *      *      *      *      (CARDS A,B,C,D,E,F ARE REQUIRED)
      *      *      *      * =5: RUN THE PREVIOUS CASE OVER BUT CHANGE THE
      *      *      *      *      PROCESS PARAMETERS AND THE PROCESS MODEL
      *      *      *      *      (CARDS D,F ARE REQUIRED)
      *      *      *      * =6: USED BY PROGRAM INTERNALLY
      *      *      *      * =7: RUN THE PREVIOUS CASE OVER BUT CHANGE THE
      *      *      *      *      PROCESS MODEL
      *      *      *      *      (CARD F REQUIRED)

```



```

*****
*
*
CARD NO.* VARIABLE * FORMAT * DISCRIPTION
*
*****
*
*
A * NI * I2 *NUMBER OF STATE VARIABLES
*
B * TOI * F10.0 *INITIAL TIME
* TMAX * F10.0 *FINAL TIME
* H * F10.0 *STEP SIZE
* PRT * F10.0 *INTEGRATION PRINT INTERVAL
*
C * XU(I) * 9F10.0 *INITIAL VALUE OF THE STATES
*
D * PK * F10.0 *PROCESS GAIN
* BETA * F10.0 *PROCESS DAMPING PARAMETER
* THETA * F10.0 *PROCESS DIMENSIONLESS DEAD-TIME
*
E * PCENT * F10.0 *OVERSHOOT CRITERIA
*
F * IMODEL * I2 *PROCESS MODEL
*****

```

```

C*****
C*****
C-----OUTPUT REGULATOR PROGRAM LISTING
C*****
C*****
      DIMENSION X(10),X0(10),DX(10),RK1(10),RK2(10),RK3(10),RK4(10)
      DIMENSION Y(500),NTITLE(15),YM(500)
      DIMENSION RTIME(5),OSHOOT(5),STIME(5)
101  FORMAT(15A4)
102  FORMAT(5I2)
104  FORMAT(1X,'T0 = ',F10.3,',',1X, 6('X0(',I2,')= ',E10.4))
105  FORMAT(1X,'RISE TIME = ',F6.3,3X,'SETTLING TIME = ',F6.3,3X,'OVERS
      HOOT = ',F6.4,3X,'IAE = ',F8.4,3X,'I = ',F8.3)
100  READ(5,102)NP,NOPL0T
      IF(NP .EQ. 0)STOP
      READ(5,101)NTITLE
19  DO 20 I=1,500
      YM(I)=0.0
20  Y(I)= 0.0
      IPLOT=0
1  J=0
      IOS = 0
      JP =0
      CALL ODE (N,M,T0,X0,M,IP,X,DX,NP,ORST,PCENT)
      IF(N .GT. 0)GO TO 3
      IF(NOPL0T .EQ. 0) GO TO 100
      GO TO 18
3  NP1V=M/IP +1
      NP1=N+1
      IPLOT=IPLOT +1
      RTIME(IPLOT) = 0.0
      OSHOOT(IPLOT) = 0.0
      STIME(IPLOT) = 0.0
      ESTIME = 0.0

```

```

      OSOTME = 0.0
      DO 13 K=1,M
      T = T0
      DO 5 I=1,N
5      X(I)=X0(I)
      IK=1
      CALL DERIV(T,X,DX,IK)
C      CHECK FOR RISE TIME , SETTLING TIME AND OVERSHOOT
      IF (IOS .GT. 0) GO TO 29
      IF (X(1)-1.0+PCENT)48,28,28
28 ESTIME = T
      IOS = 1
29 GO TO (30,32,38,38),IOS
30 IF ( DX(3) ) 31,31,48
31 RTIME(IPL0T) = T
      IOS = 2
      DX3OLD = DX(3)
      GO TO 48
32 IF ( -X(2) )36,36,34
34 OSHOOT(IPL0T) = -DX3OLD
      OSOTME = T
      IF (NP .LT. 100) GO TO 35
      ORST = OSHOOT(IPL0T)
      IPL0T = IPL0T + 1
      GO TO 1
35 IOS = 3
36 DX3OLD = DX(3)
      GO TO 48
38 Z = ABS(X(1)-1.0)
      IF(Z.GT.PCENT)GO TO 40
      IF(IOS,E0.4)GO TO 48
      STIME(IPL0T) = T
      IOS = 4
      GO TO 48

```

```

40 IF(IOS,EQ,3)GO TO 48
   STIME(IPL0T) = T
   IOS= 3
48 CONTINUE
   T =T0+(H/2.)
   DO 7 I=1,N
      RK1(I)=DX(I)*H
7    X(I)=X0(I)+(RK1(I)/2.)
      IK=2
      CALL DERIV(T,X,DX,IK)
      DO 9 I=1,N
         RK2(I)=DX(I)*H
9    X(I)=X0(I)+(RK2(I)/2.)
      IK=3
      CALL DERIV(T,X,DX,IK)
      DO 11 I=1,N
         RK3(I)=DX(I)*H
11   X(I)=X0(I)+RK3(I)
      T =T0+H
      IK=4
      CALL DERIV(T,X,DX,IK)
      IF(NOPLOT .EQ. 0)GO TO 16
      J =J-1
      IF(J,GT,0)GO TO 16
      J =IP
      JP =JP+1
      Y(JP)=T0
      YM(JP)=T0
      JPI=IPL0T*NP1V + JP
      Y(JPI)=X0(1)
      YM(JPI)=X(NP1)
C WRITE CARD SHOULD BE REPLACED HERE WITHOUT C IN COL 1 FOR OUTPUT
C   WRITE(6,104)T0,(1,X0(I),I=1,N),NP1,X(NP1)

```

```

16  T0=T0+H
    DO 13 I=1,N
      RK4(I)=DX(I)*H
13  X0(I)=X0(I)+(RK1(I)+2.*RK2(I)+2.*RK3(I)+RK4(I))/6,
    IF(OSHOOT(IPL0T) .EQ. 0.0) OSHOOT(IPL0T)=-DX(3)
    ORST = OSHOOT(IPL0T)
    IF (NP .EQ. 100) GO TO 1
    IF(STIME(IPL0T) .EQ. 0.0)ESTIME=0.0
    IF(STIME(IPL0T).LT.(1.1*OS0TME))STIME(IPL0T)=ESTIME
    WRITE(6,105)RTIME(IPL0T),STIME(IPL0T),OSHOOT(IPL0T),X(4),X(5)
    IF(IPL0T .LT. 5)GO TO 1
    IF(N0PLOT .GT. 0)GO TO 18
    IPL0T = 0
    GO TO 1
18  NDAIV=IPL0T+1
    NLP=100
    IF ( IPL0T .EQ. 0) GO TO 19
    CALL JG0LOT(Y,NP1V,NDAIV,NLP,NTITLE)
    CALL JG0LOT(YM,NP1V,NDAIV,NLP,NTITLE)
    GO TO 100
END

```

```

C*****
C-----READ INPUT AND CALCULATE DERIVATIVES
C*****
      SUBROUTINE ODE(N,H,TD,X0,M,IP,X,DX,NP,ORST,PCENT)
      DIMENSION X(10),DX(10),X0(10),XDI(10)
501  FORMAT(8F10,0)
502  FORMAT(5I2)
      800 FORMAT(/,1X,30('* *'),/,1X,'THE SECOND ORDER PLUS DEAD-TIME PROCE
      1SS PARAMETERS ARE',/,1X,'KP =',F8,3,5X,'BETA =',F8,3,5X,'THETA =',
      2F8,3,/,1X,'THE RESULTS ARE FOR A UNIT CHANGE IN SET POINT',/,1X,30
      3('* *'))
      802 FORMAT (/,1X,'THE CONTROLLER SETTINGS ARE',/,1X,'KC =',E12,4,5X,
      1'TI =',E12,4,5X,'TD1 =',E12,4,5X,'TD2 =',E12,4)
      803 FORMAT(1H1,14X,' 1 = INPUT--R-K,PLANT,REGULATOR',/,1X,'CASE = ',I2
      1,5X,' 2 = INPUT--R-K,PLANT,CONTROLLER SETTINGS',/,15X,' 3 = CHANGE
      2 VALUE OF P FOR THE CASE JUST RAN',/,15X,' 4 = SET TD2 EQUAL ZERO
      3FOR SOPDT CASE JUST RAN',/,15X,' 5 = CHG MODEL FOR CASE JUST RAN',
      4/,15X,' 6 = SET TD1 EQUAL ZERO FOR FOPDT CASE JUST RAN')
      810 FORMAT (/,1X,'PARAMETERS FOR THE **',I3,2X,'** ORDER MODEL ARE',/,
      11X,'KP =',F8,3,5X,'BETA =',F8,3,5X,'THETA =',F8,3)
C-----INPUT DATA----TYPE OF CASE TO BE RUN
      IF(NP .GT. 10)GO TO 160
      IF(NP .LT. 10)GO TO 80
      IF(KASE .LT.9)GO TO 85
      KASE = 8
      IF((IMODEL+IPOX) .EQ. 4) GO TO 113
      IF(IPOX .EQ. 2) GO TO 121
      GO TO 85
      80 READ(5,502)KASE
      IF(KASE .EQ. 0) GO TO 90
      WRITE(6,803)KASE
      85 GO TO (101,101,107,113,108,121,101,140),KASE
      90 N=0
      RETURN
C-----INPUT DATA----RUNGE-KUTTA
      101 READ(5,502)NI

```

```

      READ(5,501)T01,TMAX,H,PRT
      READ(5,501)(X01(I),I=1,N1)
C-----INPUT DATA-----SYSTEM OR PLANT
      READ(5,501)PK,B,THETA
      R = 1.0
      WRITE(6,800)PK,B,THETA
      READ(5,501)PCENT
      IF (KASE.EQ. 2) GO TO 103
C-----INPUT DATA-----LINEAR REGULATOR
      107 READ(5,501)P
      IF(KASE.EQ. 3)GO TO 102
      READ(5,501)STEP,TOL
      108 READ(5,502)IMODEL,IPOX
      IF (KASE.LT. 7) GO TO 102
      GO TO 150
      140 READ(5,501)PK,B,THETA
      IF(PK.EQ. 0.0) GO TO 90
      WRITE(6,800)PK,B,THETA
      150 NP = 100
      KASE = 9
      J = 1
      P = B*THETA
      F1 = P
      GO TO 102
      160 IF ( J.GT. 1) GO TO 165
      WRITE(6,514)P,ORST
      J = 2
      P = (1.0+10.0*(ORST-PCENT))*P
      F2 = P
      F4 = ORST
      GO TO 102
      165 F3 = F4
      WRITE(6,514)P,ORST
      514 FORMAT(1X,'P= ',F10.3,5X,'OVERSHOOT = ',F10.4)
      F4 = ORST
      IF(ORST.GT. (PCENT+1.01))GO TO 168

```

```

        IF(URST .LT. (PCENT*0.99))GO TO 168
        NP = 10
        GO TO 102
168 P = F2-((F2-F1)/(F4-F3))*(F4-PCENT)
        IF(P .LT. 0.2)P=0.2
        IF(P .GT. 100.0)P=100.0
        F1 = F2
        F2 = P
102 CALL VALUE(KASE,PK,3,THETA,P,IMODEL,STEP,TOL,CK,TI,TD1,TD2,BETAM,
        THETAM,IPOX,NP,J)
        GO TO 104
C-----INPUT DATA----CONTROL SETTINGS IF NOT CALCULATED
103 READ(5,501)CK,TI,TD1,TD2
        IMODEL=0
        BETAM=0.0
        THETAM=0.0
        GO TO 104
113 TD2=0.0
        GO TO 104
121 TD1 = 0.0
104 NDT = THETA/H + 0.49999
        IP=PRT/H + 0.4999
        N=NI
        TU=T0I
        DO 190 I=1,N
190 X0(I)=X0I(I)
        M =(TMAX-T0)/H +0.4999
        NP1=N+1
        CALL LAG (NDT,INDX)
200 U = CK*(R-X0(1))
        UNP1=0.0
        UDOT = 0.0
        FFUDOT = 0.1
        IF(NP .GT. 10) RETURN
        WRITE(6,810) IMODEL,PK,BETAM,THETAM
        TIOTAU=TI/BETAM

```



```

TD10TAU=TD1/BETAM
TD20TAU=TD2/BETAM
IF (IMODEL .GT. 1) GO TO 300
TAUOTO=BETAM/THETAM
PEN = P/(BETAM*BETAM)
WRITE(6,511)TAUOTO,T10TAU,TD10TAU,PEN
511 FORMAT(1X,'TAU/TO =',F7.4,3X,'T1/TAU =',F7.4,3X,'TD1/BETAM =',F7.4,
1/'1X,'PENALTY = ',F7.4)
GO TO 302
300 WRITE(6,512)BETAM,T10TAU,TD10TAU,TD20TAU
512 FORMAT(1X,'BETAM =',F7.4,3X,'T1/BETAM =',F7.4,3X,'TD1/BETAM =',F7.4,
14.3X,'TD2/BETAM =',F7.4)
302 WRITE(6,802)CK,T1,TD1,TD2
RETURN
C-----
ENTRY DERIV(T,X,DX,IK)
C-----
GO TO (1,2,2,4),IK
1  UPRV = U
U = CK*(H-X(1))*(X(3)/T1)-TD1*X(2)-TD2*(UNP1*PK-B*X(2)-X(1))
X(NP1)=U
CALL DEDTIM(U,NDT,INDX.UN,UNP1)
UNK = UN
UDOT = (UDOT*FFUDOT)+(1.0-FFUDOT)*(U-UPRV)/H)
GO TO 5
2  UNK = (UN+UNP1)/2.
GO TO 5
4  UNK = UNP1
5  YDDOT = PK*(UNK)-B*X(2)-X(1)
DX(1) = X(2)
DX(2) = YDDOT
DX(3) = R - X(1)
DX(4) = ABS(DX(3))
DX(5) = UX(3)*DX(3) + P*UDOT*UDOT
RETURN
END

```

```

C*****
C-----GET THE CONTROLLER SETTINGS
C*****
      SUBROUTINE VALUE(K,PK,BETA,THETA,R,IMODEL,STEP,TOL,CK,TI,TD1,TD2,
1BETAM,THETAM,IPOX,NP,J)
      DIMENSION A(10,10),B(10,10),C(10,10),S(10,10),RJ(10,10),G(10,10)
800 FORMAT(/,1X,'** K GAIN MATRIX IS **')
801 FORMAT(/,1X,'** J MATRIX IS **')
802 FORMAT(/,1X,'** STEP =',E12.4,5X,'** TOL =',E12.4)
803 FORMAT(/,1X,'** A MATRIX IS **')
804 FORMAT(/,1X,'** B MATRIX IS **')
805 FORMAT(/,1X,'** C MATRIX IS **')
806 FORMAT(/,1X,'** S MATRIX IS **')
807 FORMAT(/,1X,'** R=',E12.4)
808 FORMAT(///,1X,'THE INPUTS TO THE LINEAR REGULATOR ARE')
809 FORMAT(///,1X,'THE OUTPUTS TO THE LINEAR REGULATOR ARE')
811 FORMAT(/,1X,'** NO. OF ITERATIONS =',15)
      NDM=10
      IF(K,EO,3)GO TO 5
      IF(K,LT,8)GO TO 2
      IF(J,GT,1)GO TO 5
2  CALL MODEL(THETA,BETA,IMODEL,IPOX,THETAM,BETAM,A,B,C,S,NR,NCR,NDM)
5  IF(NP,GT,10) GO TO 6
      WRITE(6,808)
      WRITE(6,803)
      CALL WMAT(A,NR,NR,NDM)
      WRITE(6,804)
      CALL WMAT(B,NR,NCR,NDM)
      WRITE(6,805)
      CALL WMAT(C,NR,NR,NDM)
      WRITE(6,806)
      CALL WMAT(S,NR,NR,NDM)
      WRITE(6,807)R
      WRITE(6,802)STEP,TOL

```

```

6 CALL RICCAT(NR,NCB,A,B,0,S,R,STEP,TOL,RJ,G,NITER,NDM)
  IF(NP.GT. 10) GO TO 7
  WRITE(6,809)
  WRITE(6,801)
  CALL WMAT(RJ,NR,NR,NDM)
  WRITE(6,800)
  CALL WMAT(G,NCH,JR,NDM)
  WRITE(6,811)NITER
7 IF(IMODEL.GT. 1) GO TO 40
C      IPOX IS THE DEAD-TIME APPROXIMATION
C      IPOX = 1, FIRST ORDER TAYLOR SERIES EXPANSION
C      IPOX = 2, PADE APPROXIMATION
  IF(IPOX.GT. 1) GO TO 10
C      CONTROLLER SETTINGS FOR FOPDTM WITH TAYLOR APPROX
  CK = ( BETAM*G(1,2))/(PK*(1.0 + G(1,2)*THETAM))
  TI =(G(1,2)*BETAM)/(G(1,1) + G(1,2))
  TD1 =0.0
  TD2 = 0.0
  RETURN
C      CONTROLLER SETTINGS FOR FOPDTM WITH PADE APPROX
10 F1 = (1.0/BETAM)*G(1,2) + G(1,3)
  F2 = G(1,2)+((THETAM+2.0*BETAM)/2.0)*F1
  F3 = 1.0 + (THETAM*F1)/2.0
  F4 = F1 + G(1,1)
  CK = F2/(PK*F3)
  TI = F2/F4
  TD1 = (THETAM*BETAM*F1)/(2.0*F2)
  TD2 = 0.0
  RETURN
C      CONTROLLER SETTINGS FOR SOPDTM WITH TAYLOR APPROX
40 IF(IPOX.GT. 1) GO TO 50
  F1 = THETAM*G(1,2) + G(1,3)
  F2 = 1.0 + THETAM*F1

```

```

F3 = G(1,2) + BETAM*F1
F4 = G(1,1) + F1
CK = F3/(PK*F2)
TI = F3/F4
TD1 = F1/F3
TD2 = 0.0
RETURN

C          CONTROLLER SETTINGS FOR SOPDTM WITH PADE APPROX
50 F1 = G(1,3) + G(1,4)
F2 = G(1,2) + (BETAM + (THETAM/2.0))*F1
F3 = G(1,3) + (1.0 + (BETAM*THETAM)/2.0)*F1
F4 = 1.0 + (THETAM/2.0)*F1
CK = F2/(PK*F4)
TI = F2/(G(1,1) + F1)
TD1 = F3/F2
TD2 = (THETAM/2.0)*(F1/F2)
RETURN
END

```

```

C*****
C-----TRANSPORTATION LAG OR DEAD TIME SIMULATION
C*****
      SUBROUTINE LAG(N,K)
      DIMENSION A(2000)
      DO 5 I=1,N
5     A(I)=0,0
      I=N
      K=1
      RETURN
C-----
      ENTRY DEDTIM(X,V,K,XN,XNP1)
C-----
      XN=A(I)
      IP1=I+1
      IF(I.EQ.N) IP1=1
      XNP1=A(IP1)
      A(I)=X
      I=I+1
      IF(I.GT.N) I=1
      K=K+1
      RETURN
      END

```

```

C*****
C-----TRANSFORM THE SYSTEM EQUATIONS
C*****
      SUBROUTINE MODEL(THETA,BETA,IMODEL,IPOX,DTM,TAU,A,B,Q,S,NR,NCB,NDM
1)
      DIMENSION A(NDM,1),B(NDM,1),S(NDM,1),Q(NDM,1)
      NCB = 1
      NR = IMODEL+IPOX
4   DO 5 I =1,NR
      DO 5 J =1,NR
      A(I,J) = 0.0
      Q(I,J) = 0.0
5   S(I,J) = 0.0
      DO 10 I= 1,NR
      DO 10 J =1,NCB
10  B(I,J) = 0.0
      IF (IMODEL .LT. 2) GO TO 20
      DTM = THETA
      TAU = BETA
C          IPOX IS THE DEAD-TIME APPROXIMATION
C          IPOX = 1, FIRST ORDER TAYLOR SERIES EXPANSION
C          IPOX = 2, PADE APPROXIMATION
      IF(IPOX .EQ. 1) GO TO 15
      A(1,2) = 1.0
      A(2,3) = 1.0
      A(2,4) =-1.0
      A(3,1) = -2.0/DTM
      A(3,2) = -((2.0*TAU /DTM )+1.0)
      A(3,3) = -((2.0/DTM)+TAU)
      A(3,4) = ((4.0/DTM)+TAU)
      B(4,1) = 1.0
      Q(1,1) = 1.0
      RETURN

```

```

15 A(1,2) = 1.0
   A(1,3) = -DTM
   A(2,1) = -1.0
   A(2,2) = -TAU
   A(2,3) = 1.0+DTM*TAU
   F(3,1) = 1.0
   G(1,1) = 1.0
   RETURN
20 DY = 1.0
   Y = 1.0
   CALL FOPDTM( T ,DY,BETA,DTM,TAU)
   DTM = IMETA+DTM
   IF(IPOX ,E2. 1) GO TO 25
   A(1,2) = 1.0
   A(1,3) = -1.0/TAU
   A(2,1) = -2.0/(DTM*TAU)
   A(2,2) = -(DTM+2.0*TAU)/(DTM*TAU)
   A(2,3) = (DTM+4.0*TAU)/(DTM*TAU+TAU)
   B(3,1) = 1.0
   Q(1,1) = 1.0
   RETURN
25 A(1,1) = -1.0/TAU
   A(1,2) = 1.0/TAU
   B(1,1) = -DTM/TAU
   B(2,1) = 1.0
   Q(1,1) = 1.0
   RETURN
   END

```

```

C*****
C-----DETERMINE THE PARAMETERS OF THE FIRST ORDER MODEL
C*****
SUBROUTINE FOPDT4(T,DY,BETA,THETA,TAU)
  ZETA = BETA/2.0
  DELY = 0.284*DY
  IF(BETA - 2.0) 1,3,5
  1 C = SQRT(1.0-ZETA*ZETA)
    ANG = ATAN2(C,ZETA)
    I = 1
    GO TO 7
  3 I = 2
    GO TO 7
  5 C = SQRT(ZETA*ZETA-1.0)
    C4 = ZETA/C
    I = 3
    NP = 0
    6 NP = NP+1
    K = 0
    10 K = K+1
    GO TO (15,20,25),I
  15 E = 1.0/(EXP(ZETA*T))
    Y = 1.0-(E/C)*SIN(C*T+ANG)
    GO TO 40
  20 E = 1.0/(EXP(T))
    Y = 1.0-(1.0+T)*F
    GO TO 40
  25 E=1.0/(EXP((ZETA-C)*T))
    E1=(C+ZETA)*T
    IF(E1.GT. 150.)GO TO 26
    E1 = 1.0/(EXP(E1))
    Y = 1.0-0.5*E*(1.0+C4)-0.5*E1*(1.0-C4)
    GO TO 40

```



```

26 Y = 1.0-0.5*E*(1.0+C4)
40 IF(K ,GT. 1)GO TO 50
   Y2 =Y
   T2 = T
   T = 2.0*T
   GO TO 10
50 YDIFF = ABS(Y-DELY)
   IF(YDIFF ,LE.0.0002) GO TO 60
   Y1 =Y2
   T1 = T2
   Y2 = Y
   T2 = T
   T = T2-(((T2-T1)/(Y2-Y1))*(Y2-DELY))
   GO TO 10
60 IF(NP ,GT. 1) GO TO 70
   TPT03 = T
   DELY = 0.632*DY
   GO TO 8
70 TPT =T
   THETAM = 1.5*TPT03-0.5*TPT
   TAU = 1.5*(TPT-TPT03)
   RETURN
   END

```

```

C*****
C-----SOLUTION TO THE RICCATI EQUATION
C*****
      SUBROUTINE RICCAT(NR,NCB,A,B,Q,S,R,STEP,TOL,RJ,DUMMY,NITER,NDM)
      DIMENSION A(NDM,1),B(NDM,1),Q(NDM,1),S(NDM,1),RJ(NDM,1)
      DIMENSION DUMMY(NDM,1)
      DIMENSION P(10,10),RJO(10,10),W(10,10),AT(10,10),BT(10,10)
C-----CALCULATE A(TRANSP), B(TRANSP), R(INVERSE)
      CALL TRANS(A,NR,NR,AT,NDM)
      CALL TRANS(B,NR,NCB,BT,NDM)
C-----SINCE R IS A SCALAR, R(INVERSE)=1.0/R
      RI=1.0/R
C-----INITIAL VALUE OF J IS S
      NITER=0
      DO 100 I=1,NR
      DO 100 J=1,NR
      100 RJ0(I,J)=S(I,J)
C-----CALCULATE P=B*R(INVERSE)*B(TRANSP)
      CALL ADDMAT(0,A,RI,BT,NCB,NR,W,NDM)
      CALL MULMAT(B,W,NR,NCB,NR,P,NDM)
C-----CALCULATE W=Q+J*(A-P*J)+A(TRANSP)*J
      10 CALL MULMAT(P,RJO,NR,NR,NR,DUMMY,NDM)
C-----KEEP TRACK OF THE NO. OF ITERATIONS
      NITER=NITER+1
      C=-1.0
      CALL ADDMAT(1,A,C,DUMMY,NR,NR,W,NDM)
      CALL MULMAT(RJO,W,NR,NR,NR,DUMMY,NDM)
      C=1.0
      CALL ADDMAT(1,Q,C,DUMMY,NR,NR,W,NDM)
      CALL MULMAT(AT,RJO,NR,NR,NR,DUMMY,NDM)
      CALL ADDMAT(1,W,C,DUMMY,NR,NR,W,NDM)

```

```

C-----CALCULATE A NEW J BASED ON OLD J  J(I+1)=J(I)+HOW
C=STEP
CALL ADDMAT(1,RJO,C,W,NR,NR,RJ,NDM)
CALL SYMMAT(RJ,NR,NR,NDM)
C-----TOLERANCE CHECK
DO 20 I=1,NR
DO 20 J=1,NR
CHECK=ABS(RJ(I,J)-RJO(I,J))
ALLOW=ABS(RJ(I,J)*TOL)
IF(CHECK .GT. ALLOW) GO TO 29
20 CONTINUE
GO TO 50
29 IF(NITER .GE. 500)GO TO 45
30 DO 40 I=1,NR
DO 40 J=1,NR
40 RJO(I,J)=RJ(I,J)
GO TO 10
45 WRITE(6,110)
110 FORMAT(/,1X,'***FAILED TO CONVERGE * RICCATI***')
WRITE(6,111)
111 FORMAT(/,1X,'***OLD J MATRIX WAS***')
CALL WHAT(RJO,NR,NR,NDM)
C-----CALCULATE GAIN MATRIX
50 CALL MULMAT(RT,RJ,NCB,NR,NR,DUMMY,NDM)
CALL ADDMAT(0,A,RI,DUMMY,NCB,NR ,DUMMY,NJM)
RETURN
END

```

C*****

C-----READ AND WRITE A MATRIX

C*****

 SUBROUTINE RAWMAT(X,NR,NC,NDM)

 DIMENSION X(NDM,1)

 501 FORMAT(2I2)

 502 FORMAT(8F10.0)

 503 FORMAT(1X,I3,' ROWS ',I3,' COLUMNS ')

 551 FORMAT(1X,10E12.4)

 READ(5,501)NR,NC

 DO 4 I=1,NR

 4 READ(5,502)(X(I,J),J=1,NC)

C-----

 ENTRY WMAT(X,NR,NC,NDM)

C-----

 WRITE(6,503)NR,NC

 DO 8 I =1,NR

 8 WRITE(6,551)(X(I,J),J=1,NC)

 RETURN

 END

C*****

C MATRIX MULTIPLICATION

C*****

 SUBROUTINE MULMAT(A,B,NRA,NCA,NCB,R,NDM)

 DIMENSION A(NDM,1),R(NDM,1),R(NDM,1)

C-----MULTIPLY MATRIX A (TIMES) MATRIX B

 DO 10 L=1,NRA

 DO 10 K=1,NCB

 SUM = 0.0

 DO 9 I=1,NCA

 9 SUM =SUM + A(L,I)*B(I,K)

 10 R(L,K)=SUM

 RETURN

 END

```

C.....
C  MATRIX ADDITION WITH SCALAR MULTIPLICATION
C.....
      SUBROUTINE ADDMAT(M,A,C,B,NR,NC,R,NDM)
      DIMENSION A(NDM,1),B(NDM,1),R(NDM,1)
      IF (M .LT. 1) GO TO 11
C-----ADDITION WITH SCALAR MULTIPLICATION
      DO 10 I=1,NR
      DO 10 J=1,NC
      10 R(I,J) =A(I,J) + C*B(I,J)
      RETURN
C-----SCALAR MULTIPLICATION ONLY
      11 DO 20 I=1,NR
      DO 20 J=1,NC
      20 R(I,J) =C*B(I,J)
      RETURN
      END

C.....
C  A TRANSPOSE IS AT
C.....
      SUBROUTINE TRANS(A,NRA,NCA,AT,NDM)
      DIMENSION A(NDM,1),AT(NDM,1)
C-----TRANSPOSE THE MATRIX A
      DO 10 I=1,NRA
      DO 10 J=1,NCA
      10 AT(J,I)=A(I,J)
      RETURN
      END

```

```

C*****
C-----MAKE THE MATRIX SYMMETRIC
C*****
      SUBROUTINE SYMMAT(A,NR,NC,NDM)
      DIMENSION A(NDM,1)
      NRM1=NR-1
      DO 5 I=1,NRM1
      M=I+1
      DO 5 J=M,NC
      A(I,J)= (A(I,J)+A(J,I))/2,
5      A(J,I)= A(I,J)
      RETURN
      END

```

INPUT CARDS FOR OUTPUT REGULATOR PROGRAM

CARD NO.	VARIABLE	FORMAT	DISCRIPTION
1	1	1	1

```

*****
CARD NO. * VARIABLE * FORMAT * DISCRIPTION
*****
A * NI * 12 * NUMBER OF STATE VARIABLES
B * TOI * F10.0 * INITIAL TIME
  * TMAX * F10.0 * FINAL TIME
  * H * F10.0 * STEP SIZE
  * PRT * F10.0 * INTEGRATION PRINT INTERVAL
C * XO(I) * 8F10.0 * INITIAL VALUE OF THE STATES
D * PK * F10.0 * PROCESS GAIN
  * B * F10.0 * PROCESS DAMPING PARAMETER
  * THETA * F10.0 * PROCESS DIMENSIONLESS DEAD-TIME
E * PCENT * F10.0 * OVERSHOOT CRITERIA
F * P * F10.0 * PENALITY PARAMETER
G * STEP * F10.0 * REGULATOR STEP SIZE
  * TOL * F10.0 * TOLERANCE FOR THE SOLN TO THE RACCITI EQ.
H * IMODEL * 12 * PROCESS MODEL
  * IPOX L * 12 * DEAD-TIME APPROXIMATION
  * * * =1:TAYLOR
  * * * =2:PADE
I * CK * F10.0 * CONTROLLER GAIN
  * TI * F10.0 * CONTROLLER RESET TIME
  * TD1 * F10.0 * CONTROLLER PREACT TIME
  * TD2 * F10.0 * CONTROLLER PREACT TIME-SQUARED
*****

```


VITA

The author was born in New Orleans, Louisiana on September 24, 1940. He received his elementary education at St. Joseph's School in Paulina, Louisiana and his high school education at Lutchter High School in Lutchter, Louisiana, graduating in May, 1958.

His undergraduate work was at Louisiana State University and Southeastern Louisiana University, graduating with a Bachelor of Science degree in Mathematics in May, 1963. He held several positions during the six years of employment with Chrysler Corporation Space Division which followed. He returned to Louisiana State University in 1969 where he received a Master of Science degree in Chemical Engineering in 1973.

The author is married to the former Miss Barbara Hudson of Baton Rouge, Louisiana. He is the proud father of an 8 year old daughter, Lisa, and a one year old son, Stephen Troy.

He is at the present time, a candidate for the degree of Doctor of Philosophy in Chemical Engineering and is an employee of EXXON Chemical, USA.


EXAMINATION AND THESIS REPORT

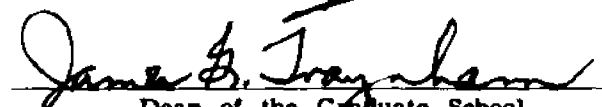
Candidate: Jacob Martin, Jr.

Major Field: Chemical Engineering


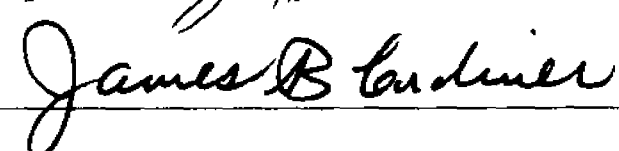
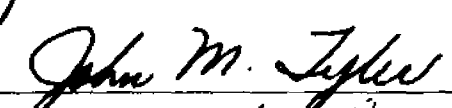
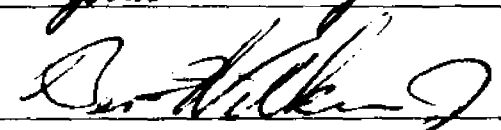
Title of Thesis: Design and Comparison of Controllers Based on Simplified Models
of the Process

Approved:


Major Professor and Chairman


Dean of the Graduate School

EXAMINING COMMITTEE:

Date of Examination:

November 21, 1975